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Thermodynamic Limit of Correlation Functions in a System of Gravitating Fermions

Bernhard Baumgartner

Institut für theoretische Physik der Universität Wien, Wien, Austria

Abstract. We show that the correlation functions in a system of gravitating fermions converge as tempered distributions in the thermodynamic limit, if the system is not at the point of phase-transition. The densities converge to the density of the Thomas-Fermi-theory and are not correlated in the limit.

I. Introduction

It has been shown by P. Hertel et al. ([1, 2]) that non-relativistic gravitating fermions have a kind of thermodynamic limit and that in the limit the system is governed by temperature-dependent Thomas-Fermi- (T.F.-) equations. What is unusual in this limit is the dependence of parameters on the particle-number N: the system is confined to a region, the linear dimensions of which vary as $N^{-1/3}$ and the temperature is set proportional to $N^{4/3}$ or the energy proportional to $N^{7/3}$, if one works with the microcanonical ensemble.) The free energy divided by $N^{7/3}$ has then a definite finite limit when N tends to infinity. To make things conceptually simpler and to obtain a certain similarity to the usual thermodynamic formulas, we transform the Hamiltonian

$$H_N = \sum_{i=1}^{N} p_i^2/2m - \kappa \sum_{i<j} |x_i - x_j|^{-1} \tag{1}$$

with the unitary transformation

$$x \mapsto N^{-1/3} x,\ p \mapsto N^{1/3} p \tag{2}$$

and divide it by $N^{4/3}$.

The resulting Hamiltonian,

$$\tilde{H}_N = N^{-2/3} \sum_{i=1}^{N} p_i^2/2m - \kappa/N \sum_{i<j} |x_i - x_j|^{-1} \tag{3}$$

with Dirichlet boundary-conditions in $L^2(V)$, (V does not depend on N) serves to define a canonical ensemble with temperature β^{-1} (which is also N-independent).