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Heat Equation on Phase Space and the Classical Limit of Quantum Mechanical Expectation Values

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Abstract. The expectation value of a quantum mechanical operator, taken in coherent states and suitably rescaled, is the solution of an initial value problem for the heat equation on phase space, in which $\hbar$ plays the role of time, and the classical observable is the distribution of temperature at $\hbar=0$.

Introduction

A recent paper by Hepp [1] is devoted to the classical limit of (rescaled) expectation values in coherent states and to their time evolution. Here we sharpen some results of [1] by relating the classical limit to an initial value problem in $\hbar$. This is done with the help of a quantization formula derived in [2].

Notations

Denote by E a 2ν-dimensional real vector space with a symplectic form σ. (Phase space for $\nu<\infty$ degrees of freedom.) Elements of E will be denoted by $a, b, v\dots$. Fix on E a σ-allowed complex structure J, i.e. a linear map satisfying $J^2=-1$, $\sigma(Ja, Jv)=\sigma(a, v)$ and $\sigma(a, Ja)>0$ for $a\neq 0$. Introduce the orthogonal form $s(a, v)=\sigma(a, Jv)$, and the (phase space) Gaussian $\Omega(v)=e^{-\pi s(v, v)}$. Normalize the invariant measure dv on E by the requirement $\int \Omega(v)dv=1$. This is equivalent to the requirement $F^2=1$ where F is the symplectic Fourier transform:

$$Ff)(v)=\tilde{f}(v)=\int e^{2i\pi\sigma(v, v')}f(v')dv' .$$

In the Hilbert space $L^2(E; dv)$ consider the family of functions Ω^a:

$$\Omega^a(v)=e^{-2i\pi\sigma(a, v)}\Omega(v+a) .$$

Denote by $\mathscr{H}$ the closed linear span of the family Ω^a, with the scalar product inherited from $L^2(E; dv)$. For any $\Phi\in\mathscr{H}$ one has $(\Omega^a, \Phi)=k\Phi(-a)$, with

$$k=(\Omega, \Omega)=2^{-\nu} .$$