Commun. math. Phys. 48, 191—194 (1976)

Communications in
Mathematical
Physics

Parity Operator and Quantization of δ-Functions

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Abstract. In the Weyl quantization scheme, the δ-function at the origin of phase space corresponds to the parity operator. The quantization of a function $f(v)$ on phase space is the operator $\int f(v/2)W(v)dvM$, where M is the parity and $W(v)$ the Weyl operator.

Introduction

We are concerned here with the elementary problem of writing down an operator $Q(f)$ which quantizes a function f on (flat) phase space. The existing solutions [1] (see also [2]) all involve, to the best of our knowledge, the performing of Fourier transforms. By contrast, our equation (10 bis) picks up local contribution from the classical function and also exhibits a rather unexpected role played by the parity operator.

1. Displaced Parity Operators

Let E be the phase space for $\nu<\infty$ degrees of freedom, i.e. a 2ν-dimensional vector space over $\mathbb{R}$, with a symplectic form $\sigma(v, a)\cdot(a, v\in E)$. Let $v\rightarrow W(v)\ (v\in E)$ be a Weyl system over E, i.e. a strongly continuous family of unitary operators acting irreducibly on a separable Hilbert space $\mathscr{H}$ and satisfying

$$W(a)W(v)=e^{a}(v)W(a+v)\,. \tag{1}$$

We have introduced the abbreviation

$$e^{a}(v)=e^{2i\pi\sigma(a,\,v)}\,. \tag{2}$$

The family $W'(v)=W(-v)$ also satisfies (1). By the uniqueness theorem of von Neumann, there exists in $\mathscr{H}$ a unitary operator M, determined up to a phase, and such that $W(v)M=MW(-v)$ for every $v\in E$. Since M^2 commutes with the irreducible family of operators $W(v)$, it is a number of modulus 1, which can be adjusted to 1 by a multiplication of a suitable number $e^{i\theta}$ to M. Then $M=M^*$ and M is determined up to a sign.