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Global Properties of Radial Wave Functions in Schwarzschild's Space-Time

I. The Regular Singular Points

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Abstract. The radial factor $R(x, x_s)$ of a scalar field in Schwarzschild's space-time satisfies a second order ordinary differential equation with two regular singular points at $x=0$ and $x=x_s$ and one irregular singular point at $x=\infty$. The analytical properties of four solutions $\mathscr{R}_1$, $\mathscr{R}_2$, $\mathscr{R}_3$, and $\mathscr{R}_4$ (defined by their power series expansions about $x=0$ and $x=x_s$) with respect to x_s are studied. An analytical continuation is given for each solution outside its circle of convergence. Relations to the flat-space solutions are established. Finally the coefficients relating linearly any three of these solutions are determined and studied as functions of the parameter x_s.

1. Introduction

Physical phenomena around black holes are being studied recently with increasing interest and success. The most rigorous of these studies refer to weak fields imposed on a known curved background, usually the Schwarzschild or Kerr space-time. The pattern followed in solving such problems is familiar. A numerical or approximate study [1–3] indicates the answers and then analytical methods are used to establish rigorously the results. Combinations of numerical and analytical methods [4–9] have been used in cases where some of the answers can be established rigorously and some cannot.

The obstacles preventing a rigorous analytical treatment of perturbation phenomena in curved space-times are related directly to the procedure followed in such studies. Since after linearization the essence of the problem is contained in a second order linear partial differential equation [10–12], we have to separate the partial differential equation into ordinary differential equations using the method of separation of variables and then try to solve the ordinary differential equations. The separation in Schwarzschild's space-time is simple in all cases of scalar, electromagnetic and gravitational radiation. In Kerr's space-time Carter [13, 14] and Teukolsky [15, 16] have succeeded in separating the original partial differential equation into ordinary differential equations with independent