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On the Discrete Spectrum of the Schrödinger Operators of Multiparticle Systems

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Abstract. It is shown that the discrete spectrum of the n-particle Schrödinger operators in the center-of-mass frame is finite for short-range potentials.

Section 1

1. The general structure of the spectrum of Schrödinger operators of multiparticle systems, namely the location of the essential spectrum, was determined in [1, 2] (see also [3]). These results were subsequently improved by many authors (see [4–7] and References in these articles). Further and exhaustive information on the essential spectrum of the Schrödinger operator of a multiparticle system has been obtained in scattering theory [8]. Thus, the fundamental problem in this field is now to investigate the point spectrum and, in particular, the discrete spectrum[1].

2. It was shown in [1, 2] that the discrete spectrum of a Schrödinger operator is at most countable a set whose unique point of accumulation (if any) is the infimum of the continuous spectrum (for determination of this point, see below). The Schrödinger operators of atom-type systems have infinite discrete spectra [1]. Thus, the basic problem in a qualitative description of the discrete spectrum of the Schrödinger operator is to find classes of potentials for which the discrete spectrum is finite or infinite[2]. A solution to this problem is also important for scattering theory (see [9–11]), stability and spectra theory of quantum systems.

3. Familiar examples in physics indicate that one such class is apparently that of the so-called short-range potentials, i.e., potentials which decrease sufficiently rapidly at infinity.

There is already a considerable literature devoted to the proof that the discrete spectrum of the Schrödinger operator for short-range potentials is finite. The most complete results in this area may be found in [12–14]. These papers establish

[1] The set of isolated eigenvalues of finite multiplicity is called here the discrete spectrum. The point spectrum is the set of all eigenvalues of finite multiplicity.

[2] A useful discussion of these questions can be found in [16].