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On the Generators of Quantum Dynamical Semigroups

G. Lindblad

Department of Theoretical Physics, Royal Institute of Technology,
S-100 44 Stockholm 70, Sweden

Abstract. The notion of a quantum dynamical semigroup is defined using the concept of a completely positive map. An explicit form of a bounded generator of such a semigroup on $B(\mathscr{H})$ is derived. This is a quantum analogue of the Lévy-Khinchin formula. As a result the general form of a large class of Markovian quantum-mechanical master equations is obtained.

1. Introduction

The dynamics of a finite closed quantum system is conventionally represented by a one-parameter group of unitary transformations in Hilbert space. This formalism makes it difficult to describe irreversible processes like the decay of unstable particles, approach to thermodynamic equilibrium and measurement processes [1–3].

It seems that the only possibility of introducing an irreversible behaviour in a finite system is to avoid the unitary time development altogether by considering non-Hamiltonian systems. One way of doing this is by postulating an interaction of the considered system S with an external system R like a heat bath or a measuring instrument. This approach is suggested by the theory of the measurement process in quantum theory, which provides an example of an irreversible process even in the axioms of quantum theory, and by Einstein's theory of Brownian motion where the fluid provides a stochastic external force which determines the irreversible nature of the motion. A different physical interpretation with the same mathematical structure is to consider S as a limited set of (macroscopic) degrees of freedom of a large system $S+R$ and R as the uncontrolled (microscopic) degrees of freedom. If the reservoir R is supposed to be finite (but large) then the development of the system $S+R$ may be given by a unitary group of transformations. The partial state of S then suffers a time development which is not given by a unitary transformation in general.

The simplest dynamics for S which could describe a genuinely irreversible process is a semigroup of transformations which introduces a preferred direction