Commun. math. Phys. 48, 85—88 (1976)

Communications in
Mathematical
Physics

A Definition of Gibbs State for a Compact Set with Z^ν Action

D. Capocaccia

Istituto di Matematica, Università di Camerino, Camerino, and
GNSM-CNR, Istituto di Fisica, Università di Roma, Roma, Italy

Abstract. The definition of Gibbs states used in the equilibrium statistical mechanics of lattice spin systems is extended to apply to a compact metrizable space, where Z^ν acts by an expansive group of homeomorphisms.

Introduction

In this paper we give an extension of the definition of Gibbs states used in the equilibrium statistical mechanics of classical lattice spin systems [1]. The assumptions, definitions and results are stated in Section 1, the proofs are given in Section 3, and examples are discussed in Section 2. The discussion of the example of the toral diffeomorphism could be generalized to apply to the basic sets for diffeomorphisms satisfying Smale's Axiom A [2].

1. Results

1.1. Assumptions. Let Ω be a compact metrizable space, and $\{T^k, k\in Z^\nu\}$ a group of homeomorphisms of Ω, *expansive* with expansive constant γ. That is $\gamma>0$ is such that if $x, y\in\Omega$, and $d(T^k x, T^k y)\leqq\gamma\,\forall k\in Z^\nu$, then $x=y$.

1.2. Definitions. Two points $x, y\in\Omega$ are *conjugate* if $d(T^k x, T^k y)\xrightarrow[|k|\to\infty]{}0$.

Let $O\subset\Omega$ be open: a mapping $\varphi:O\to\Omega$ is *conjugating* if $d(T^k x, T^k\varphi(x))\xrightarrow[|k|\to\infty]{}0$ uniformely with respect to $x\in O$.

1.3. Theorem. *Suppose that for every pair of conjugate points $x, y\in\Omega$ there is an open set $O\subset\Omega$, $O\ni x$ and a mapping $\varphi:O\to\Omega$ conjugating, continous at x and such that $\varphi(x)=y$.*

Then for every such mapping one can find an open set $\tilde{O}\ni x$, $\tilde{O}\subset O$ such that φ is a homeomorphism of $\tilde{O}$ to $\varphi(\tilde{O})$. If φ' is a mapping with the same properties as φ, φ and φ' coincide on some neighbourhood of x.

1.4. Assumption. From now on we assume that the condition of the Theorem hold.