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On the Spinor Rank of Fermi Fields

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Abstract. We show that any Wightman field satisfying equal-time anti-commutation relations involving space derivatives of degree at most r must have spinor rank $r+1$.

Let ψ be a Wightman Fermi field, transforming according to the representation $\mathscr{D}_{j,k}$ of SL(2, $\mathbb{C}$):

$$U(A)\psi_{(\mu)}(x)U(A)^{-1}=(\underbrace{A^{-1}\otimes\ldots\otimes A^{-1}}_{2j}\otimes\underbrace{A^{*-1}\otimes\ldots\otimes A^{*-1}}_{2k})_{(\mu)(\nu)}\psi_{(\nu)}(\Lambda(A)x)$$

where $A\to\Lambda(A)$ is the usual homomorphism from SL(2, $\mathbb{C}$) to the Lorentz group. Suppose also that ψ satisfies canonical anti-commutation relations at time zero in the form

$$\{\psi_{(\mu)}(0,\boldsymbol{x}),\psi^{*}_{(\nu)}(0,\boldsymbol{y})\}=P_{(\mu)(\nu)}(\boldsymbol{\nabla})\,\delta^{3}(\boldsymbol{x}-\boldsymbol{y})\,. \tag{1}$$

Here, P is a polynomial of degree r, and (μ), (ν) denote spinor indices, $2j$ of which are undotted and $2k$ of which are dotted[1]. We note that free fields of spin 1/2, 3/2, ... obey such relations, with $r=0,2,\ldots$. From positivity, r must be even. If the spinor rank of ψ were $\leqq r-1$, then the left hand side of (1) would transform as a spinor of rank at most $2r-2$, i.e. the right hand side would be a polynomial in $\boldsymbol{\nabla}$ of degree at most $r-1$, a contradiction. Hence it is enough to show that the spinor rank $s=2(j+k)$, cannot exceed $r+1$, as it is odd by the spin-statistics theorem. We use the methods of [1].

Let $A=A(\lambda)\in\mathrm{SL}(2,\mathbb{C})$ be of the special form

$$A=A^{*}=\begin{pmatrix}\sqrt{\lambda} & 0\\ 0 & 1/\sqrt{\lambda}\end{pmatrix},\qquad \lambda>0$$

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1 We denote dotted indices by dashed symbols.