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Canonical and Grand Canonical Gibbs States for Continuum Systems

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Abstract. It is shown that for a large class of interactions any canonical Gibbs state satisfying a natural temperedness condition is a mixture of Gibbs states with appropriate activities, and vice versa. Some general results on Gibbs states and canonical Gibbs states are established. In particular, a differential characterization of Gibbs states is given.

0. Introduction

A state of a many particle system is called a canonical or a (grand canonical) Gibbs state[1] if its conditional probabilities in bounded volumes are given by the canonical or grand canonical Gibbs distributions, respectively. While the Gibbs states are easier to deal with, the natural candidates for the invariant states under the motion of interacting particles are the canonical Gibbs states. This intuition has been confirmed not only for lattice systems (see [15] and the references in [6]) but also for continuum systems – the relation between canonical Gibbs states and the so-called classical KMS condition recently established by Aizenmann, Goldstein, and Lebowitz[2] (private communication) is a result in this spirit.

The first question concerning canonical Gibbs states is whether they are mixtures of Gibbs states. For lattice systems an affirmative answer has been given by Thompson, Logan, Shiga, and the author (see [6, 7, 15] and the references there), and for continuum systems of independent particles by Nguyen and Zessin [11]. In this paper we do the same for continuum systems of interacting particles. The essential ideas are those of [7], but the technical details are rather different.

Now we describe the main result for the special case of shift invariant interactions.

(0.1) Theorem. *Suppose that the interaction is given by a translationally invariant finite range potential of one of the following four types:*
(PP) *Positive pair potential.*

[1] Gibbs states are often called equilibrium states satisfying the DLR-equations.
[2] Their interest in canonical Gibbs states for continuum systems stimulated the work presented here.