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New Super-Selection Sectors ("Soliton-States") in Two Dimensional Bose Quantum Field Models

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Abstract. A rigorous construction of new super-selection sectors – so-called "soliton-sectors" – for the quantum "sine-Gordon" equation and the $(\boldsymbol{\phi}\cdot\boldsymbol{\phi})^2$-quantum field models with explicitly broken isospin symmetry in two space-time dimensions is presented. These sectors are eigenspaces of the charge $\boldsymbol{Q}\equiv\int dx(\mathrm{grad}\,\boldsymbol{\phi})(x)$ with non-zero eigenvalue. The scattering theory for quantum solitons is briefly discussed and shown to have consequences for the physics in the vacuum sector. A general theory is developed which explains why soliton-sectors may exist for theories in two but not in four space-time dimensions except possibly for non-abelian Yang-Mills theories.

In quantum field theory a great deal of attention has recently been paid to the construction and analysis of new super-selection sectors orthogonal to the vacuum sector. Most authors – and this is not an accident (see Section 6) – have studied Bose quantum field models in *two* space-time dimensions such as the quantum "sine-Gordon" equation [3, 8, 14, 15] and the ϕ^4-model [4, 8, 21, 35][1] which are known to exist and to define relativistic quantum field theories [12, 36, 15, 17]. A deep axiomatic analysis of super-selection sectors in the framework of algebras of local observables has earlier been presented in [9]. (Some of the results of [9], e.g. the analysis of the statistics of a super-selection sector, do however not apply to two space-time dimensions.)

For the two dimensional models these new sectors are expected to contain states describing somewhat unusual collective phenomena which may be related to the "soliton"-solutions of the c-number, non-linear partial differential field equations, the "sine-Gordon" equation [8, 14], or the equation

$$(\square+m^2)\phi(x,t)=-\lambda\phi(x,t)^3,\; m^2<0\,;$$

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1 See also Remark 7, Section 6.