Commun. math. Phys. 47, 233—268 (1976)

Communications in Mathematical Physics

Classical and Quantum Statistical Mechanics in One and Two Dimensions: Two-Component Yukawa — and Coulomb Systems

Jürg Fröhlich

Department of Mathematics, Princeton University, Princeton, N. J. 08540, USA

Abstract. We estimate the canonical and grand canonical partition function in a finite volume and prove stability and existence of the thermodynamic limit for the pressure of two component classical and quantum systems of particles with charge $\pm\varepsilon$ interacting via two body Yukawa – or Coulomb forces. In the case of Coulomb forces we require neutrality. For the classical system in two dimensions there exists a critical temperature T_c at and below which the system collapses. For the classical Yukawa system the correlation functions exist for arbitrary fugacity and the general structure of the pure phases can be analyzed completely.

1. Introduction

a) Definition of the Problem; Connections to Euclidean Field Theory

In this paper we construct the thermodynamic limit of the pressure for systems of classical point particles in two space dimensions with charge $\pm\varepsilon$ interacting via Yukawa – or Coulomb – two body forces (or forces which can be "dominated" by these). The same result for the analogous systems in one dimension [6] and for the corresponding quantum systems in (one and) two dimensions then follow as rather simple corollaries, at least if one borrows certain estimates and techniques from [19] for the analysis of the quantum systems.

The two dimensional classical systems we are considering are *not* H-stable [26], the *classical* Hamilton function is not bounded below, and we should therefore not be surprised that there exists a critical temperature T_c depending on the strength of the forces, i.e. the charge ε, at and below which these classical systems collapse. This is due to the lack of a sufficiently strong centrifugal barrier below T_c. Also notice that the Coulomb potential in one and two dimensions is extremely long range ($\sim r$, $\log r$, respectively, where r is the distance between two charges). It is therefore not tempered. If the total charge is 0 there is screening, and the long range character of the Coulomb forces disappears. That is why we only consider neutral Coulomb systems in this paper. Results on the *classical*