Commun. math. Phys. 47, 167—170 (1976)

Communications in
Mathematical
Physics

On Uniqueness of KMS States of One-dimensional Quantum Lattice Systems

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Abstract. We present a proof of the theorem on the uniqueness of KMS states of one-dimensional quantum lattice systems, which is based on some equicontinuity.

1. Introduction

Araki [1] has proved, in full generality, the uniqueness of KMS states of one-dimensional quantum lattice systems under the condition that for some increasing family of finite volumes the corresponding surface energies are bounded. (See also [8, 3, 5, 9].) We present another proof of this fact in the same setting as in [1, 9]. The reader is referred to [1] for the connection with one-dimensional lattice systems.

2. Theorem

Let A be a UHF algebra and δ a normal *-derivation on A, i.e., the domain $D(\delta)$ of δ is the union of an increasing family $\{A_n\}$ of finite type I factors (which is dense in A). There exists $h_n = h_n^* \in A$ for each n satisfying $\delta(a) = \delta_{ih_n}(a) \equiv [ih_n, a]$ for all $a \in A_n$. Let τ be the unique tracial state on A and P_n the canonical conditional expectation of A onto A_n, i.e., $k_n \equiv P_n h_n \in A_n$ satisfies $\tau(h_n a) = \tau(k_n a)$ for all $a \in A_n$. If $\{\|h_n - k_n\|\}$ is bounded, the closure of δ generates a one parameter automorphism group ϱ_t satisfying

$$\varrho_t(X) = \lim e^{ik_n t} x e^{-ik_n t}, \qquad x \in A .$$

(For the proof, see [6].) Since ϱ_t is approximately inner, there exists at least one KMS state for any temperature [7]. On the uniqueness of KMS states we have

Theorem. *If $\{\|h_n - k_n\|\}$ is uniformly bounded, then A has only one ϱ_t-KMS state for each inverse temperature β.*

3. Proof

Let ψ be an extremal KMS state at β and $(\mathfrak{H}, \pi, \Psi)$ the GNS representation of A associated with ψ. Then Ψ is a cyclic and separating vector relative to $\mathfrak{M} \equiv \pi(A)''$.