Commun. math. Phys. 47, 155—166 (1976)

Communications in
Mathematical
Physics

Cluster Properties of Lattice and Continuous Systems

M. Duneau and B. Souillard

Centre de Physique Théorique*, Ecole Polytechnique, F-91120 Palaiseau, France

Abstract. Various strong decay properties are proved for lattice systems with general n-body interactions, and for continuous systems with two-body and n-body interactions. The range of the potentials is finite or infinite.

I. Introduction

1. Definitions [1, 2]*:*

We say that the truncated correlation functions ϱ_Λ^T satisfy *a strong cluster property (S.C.P.)* if there exists a real integrable function U of the configuration space $\mathbb{R}^\nu$ or $\mathbb{Z}^\nu$ such that for any configuration X (except perhaps a set of zero measure):

$$|\varrho_\Lambda^T(X)| \leqq A \sum_{T \in \mathfrak{T}(X)} \prod_{(x,x') \in T} U(x,x') \tag{1}$$

where the sum $\sum$ runs over all trees T on X (i.e. connected graphs without closed loop), and the product runs over all lines (x, x') of the tree T; A and U are independent of the box Λ, of X and of the number of points $|X|$ of X, but depend on the potential Φ (including here the reciprocal temperature β) and on the activity z.

In the case of a lattice system, an equivalent formulation of S.C.P. can be given (for equivalence see Appendix).

$$|\varrho_\Lambda^T(X)| \leqq A C^{|X|} \mathfrak{N}(X) e^{-L_\delta(X)} \tag{2}$$

where $\mathfrak{N}(X)$ is a numerical factor equal to $N_1! \dots N_p!$ when the points of X occupy only p different positions occuring respectively $N_1, \dots, N_p$ times, C is a constant and $L_\delta(X)$ is the shortest length with respect to some distance δ of all the trees constructed on the points of X and arbitrary other points (for example $\delta(x,x') = \chi|x-x'|$ or $\delta(x,x') = s\log(1+\alpha|x-x'|)$, $s > \nu$), with $e^{-\delta(x,x')}$ integrable with respect to x'; A, C, and δ are again independent of Λ, X, and $|X|$ but depend on Φ and z.

Moreover the truncated correlations ϱ_Λ^T are said to satisfy *a strong decrease property (S.D.P.)* if a bound of the type (1) holds, with a function $U(x-x')$ which is not integrable, or (2) with $s \leqq \nu$ or with a further multiplicative factor worse than $C^{|X|}$ (for instance $|X|!$).

In a large number of situations with two-body potentials, S.C.P. have been proved [2, 3] to be equivalent to analyticity with respect to the activities (plus

* Equipe de Recherche du C.N.R.S. 174.