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There are No Scalar Lie Fields in Three or More Dimensional Space-Time

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Abstract. We show that a Lie field structure is incompatible with a scalar relativistic quantum field theory if the dimension of space-time is greater than two. Our main argument is based on the Jacobi identity and the spectrum condition.

I. Introduction

In 1961 Greenberg [1] suggested to consider fields, for which the commutator in addition to a C-number term is linear in the field. Because of this property they were called Lie fields. In the neutral scalar case this would mean

$$[A(x), A(y)] = \Delta(x, y) + \int dz C(x, y, z) A(z) \tag{1.1}$$

where Δ and C are generalized functions.

Łopuszański [2] remarked that a Lie field theory is soluble in the sense that the Wightman functions are determined by the specification of Δ and C (see also [3]). Lehmann (see [3]) gave a simple example of a Lie field in two dimensions, namely

$$A(x) = \varphi(x) + \lambda :\varphi^2:(x) \tag{1.2}$$

where $\varphi(x)$ is a free field of mass m. This is not true in three or more dimensions. Robinson [4] claimed that Lie fields are not possible in more than two dimensions. But his argument was not entirely valid as has been shown by Lowenstein [5]. Lowenstein gave some examples of nonvanishing $C(x, y, z)$ which satisfy the Jacobi identity and are Poincaré invariant. In Section III we shall show that these examples cannot arise from a Wightman theory. Glaser (see [3]) and Greenberg [6] considered more general commutators

$$[A_i(x), A_j(y)] = \Delta_{ij}(x, y) + \sum_{k=1}^{N} \int dz C_{ijk}(x, y, z) A_k(z) \tag{1.3}$$

with N either finite of infinite and proved that even under these circumstances Lie fields cannot lead to any scattering.

In Section II we list some algebraic properties of Lie fields. Section III contains a reduction of the proof to a simple statement about the Fourier transform of the four-point function. Our main theorem about nonexistence of Lie fields in three or more dimensions will be proved in Section IV. Finally we explain why in two dimensions Lie fields are possible.

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