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Distributional Limits of Renormalized Feynman Integrals with Zero-Mass Denominators

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Abstract. It is shown that the $\varepsilon \to 0$ limits of renormalized Feynman integrals exist and define Lorentz invariant tempered distributions in the external momenta. The proof applies to the case where some or all particle masses vanish.

Within the Bogoliubov-Parsiuk-Hepp-Zimmermann (BPHZ) framework of renormalized perturbation theory [1–3], the connected Green functions of elementary and composite fields are expressed as sums of contributions from Feynman diagrams, each of which corresponds to a subtracted momentum-space integral of the form

$$J_\varepsilon(p) = \int_{\mathbb{R}^{4M}} dk R_\varepsilon(p, k) \tag{1}$$

$p=(p_1, p_2, \ldots, p_N)=$ independent external momenta ($p_i \in \mathbb{R}^4$), $k=(k_1, k_2, \ldots, k_N)=$ independent internal (loop) momenta ($k_j \in \mathbb{R}^4$).

With Zimmermann's subtraction prescription [3], if all mass parameters are positive, the integral (1) converges absolutely for all $\varepsilon > 0$; moreover, as ε tends to zero, $J_\varepsilon(p)$ approaches, in the sense of tempered distributions, a Lorentz invariant limit [4]. Zimmermann's proof of the distributional limit was based on an earlier theorem of Hepp [2]. There, also, the non-vanishing of all masses was a crucial hypothesis.

In Ref. [5], one of us (J.H.L.) introduces a modified subtraction scheme such that the integral

$$T_\varepsilon(\phi) = \int_{\mathbb{R}^{4N}} \int_{\times\, \mathbb{R}^{4M}} dp\, dk\, \phi(p) R_\varepsilon(p, k) \tag{2}$$

converges absolutely for arbitrary $\varepsilon > 0$ and $\phi \in \mathscr{S}(\mathbb{R}^{4N})$, provided that a certain infrared power-counting criterion is fulfilled. There is no requirement that any of the masses of the unsubtracted integrand be positive (there is, however, at least one non-zero normalization mass appearing in subtractions terms). In the present article, we use the absolute convergence of (2) to show that T_ε approaches, when ε tends to zero, a Lorentz invariant limit as a tempered distribution. Again, some

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