Commun. math. Phys. 46, 263—268 (1976)

Communications in
Mathematical
Physics

On the *b*-Boundary of the Closed Friedman-Model

B. Bosshard

Institut für theoretische Physik, Universität Bern, CH-3012 Bern, Switzerland

Abstract. Some points of the past Big Bang in the closed fourdimensional Friedman-model are found to be identical with points of the future collapse according to the bundle-boundary definition.

1. Introduction

Consider the closed Friedman-model (M, g) with metric

$$ds_g^2 = R^2(\psi)\{d\psi^2 - d\sigma^2 - \sin^2\sigma(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\}$$
$$\text{with } R(\psi) = 1 - \cos\psi\,,$$

with singularities at $\psi = 0$ and $\psi = 2\pi$. We shall investigate the structure of the b-boundary for this space-time by working with, rather than the ten-dimensional orthonormal bundle $O(M)$ (see [1, 2]), a certain three-dimensional subbundle. The construction is as follows. Consider the timelike and totally geodesic two-dimensional submanifolds $N c M$ with induced metric γ, given by

$$\vartheta = \text{const} \quad \text{and} \quad \varphi = \text{const}\,.$$

Moreover, there exists an orthonormal dyad field

$$W_\alpha, \quad \alpha = 2, 3$$

which is parallel along and orthogonal to N. Therefore we can construct a three-dimensional submanifold $\tilde{N} c O(M)$, consisting of every orthonormal tetrad Y_i, $i = 0, \ldots 3$ with

$$Y_A \in T(N) \quad A = 0, 1$$
$$Y_\alpha = W_\alpha \quad \alpha = 2, 3$$

at every point of N. $\tilde{N}$ is isomorphic to $O(N)$. Furthermore the induced metric in $\tilde{N}$ is equal to the bundle metric $\tilde{\gamma}$ in $O(N)$, because any curve in N, which is horizontal with respect to γ is horizontal with respect to g as well. The metric $\tilde{\gamma}$ can be easily computed. This reduction method can be applied also to other space-times, e.g. the Schwarzschild and Reissner-Nordström space-times. If we now find curves, which connect two points in the fibres of the two singularities with arbitrarily small length in $\overline{O(N)'}$ [1], the Cauchy completion of $O(N)'$ [1] [1],

[1] The prime denotes the connected component, i.e. here the manifolds consisting of every positively oriented orthonormal dyad resp. tetrad in every point of N res. M.