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Correlation Inequalities in Quantum Statistical Mechanics and Their Application in the Kondo Problem

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Abstract. We consider a large class of models which share the essential features of the Kondo model. Bounds on the susceptibility of the impurity spin are derived as consequences of general inequalities for quantum correlation functions. We also obtain bounds for the spin polarization in the presence of an external field.

1. Introduction

The Kondo model for the interaction of the conduction electrons with localized magnetic moments, in its most idealized version, concerns an isolated spin immersed in an electron gas. The purpose of the present work is to investigate the behavior of a single spin coupled to a heat bath and, in particular, to place lower and upper bounds on the susceptibility χ as a function of the temperature T and the coupling constant J. From these bounds it is seen that, as $J \to 0$, the deviation of χ^{-1} from the Curie law tends to zero *uniformly* in T. This result refutes the singular T-dependence of χ^{-1} obtained from the Kondo model in perturbation theory [1, 2], but it does not contradict $\chi(T=0)$ being finite.

First, we clarify terminology and notation. Let $\langle \cdot \rangle_{\beta H}$ denote the thermal average with respect to the hamiltonian H and the inverse temperature $\beta = 1/kT$. For any two operators A and B, Bogoliubov [3] introduced the inner product

$$(A, B) = \int_0^\beta d\lambda \langle e^{\lambda H} A^* e^{-\lambda H} B \rangle_{\beta H}$$

with the remarkable property

$$(A, B) = (B^*, A^*) .$$

The physical significance of this inner product becomes apparent if A and B are chosen to be selfadjoint and if by chance the hamiltonian contains a term $-xB$. Then

$$d/dx \langle A \rangle_{\beta H} = (A, B) - \beta \langle A \rangle_{\beta H} \langle B \rangle_{\beta H} .$$

A similar formula valid in classical statistical mechanics suggests to call $\beta^{-1}(A, B)$ the canonical correlation [4] between A and B. In accordance with this notion, the observables A and B are said to be uncorrelated (for fixed β and H) if

$$\beta^{-1}(A, B) = \langle A \rangle_{\beta H} \langle B \rangle_{\beta H}$$

meaning that the thermal average $\langle A \rangle_{\beta H}$ is invariant under an infinitesimal change $H \to H - xB$.