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First Order Phase Transition in the Plane Rotator Ferromagnetic Model in Two Dimensions

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Abstract. We show that the two-dimensional isotropic ferromagnetic rotator model exhibits a first order phase transition if the interaction decays as $r^{-\alpha}$ with $2<\alpha<4$.

Introduction

It is known that the isotropic two-dimensional ferromagnetic Heisenberg model does not exhibit spontaneous magnetization in two dimensions if the forces between the spins are not too long ranged. Typically for a potential of the type $J(r)\sim(r+1)^{-\alpha}$, where r is the distance between the two spins in interaction, we need $\alpha>2$ in order to obtain a normal thermodynamic behaviour and $\alpha>4$ for the absence of spontaneous magnetization (Mermin and Wagner [1], Ruelle [2]. This result equally holds for a variety of classical spin systems, most notably the plane rotator and the classical Heisenberg model (Mermin [3], Vuillermot, Romerio [4], Dobrushin, Shlosman [5]). These proofs have put on a firm ground already existing intuitive arguments based on the droplet model of condensation (Fisher [6], Mermin [7]). These arguments are based on the fact that in order to create a droplet D of size L of the opposite phase, one needs an energy of the order of $\sum_{r\in D} r^2 J(r)$ at worst. Therefore, if $J(r)\sim r^{-\alpha}$, r being large, we obtain three different cases depending on the value of α. If $\alpha>4$, the quantity $\sum_{r\in D} r^2 J(r)$ is always bounded by a number independent from the size of D and thus making big droplets very probable. Whereas if $\alpha\leqq 4$,

$$\sum_{r\in D} r^2 J(r)\sim L^{4-\alpha}, \qquad \alpha<4,$$

$$\sum_{r\in D} r^2 J(r)\sim \ln L, \qquad \alpha=4.$$

Therefore the energy of a droplet increases with the size of the droplet if $\alpha\leqq 4$, but as a power law when $\alpha<4$ and only logarithmically when $\alpha=4$. Such big droplets are very unlikely and therefore the order in the system cannot be destroyed, at least when $\alpha<4$. The case $\alpha=4$ is evidently more delicate. This situa-

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