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A Canonical Structure for Classical Field Theories

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Abstract. A general scheme of constructing a canonical structure (i.e. Poisson bracket, canonical fields) in classical field theories is proposed. The theory is manifestly independent of the particular choice of an initial space-like surface in space-time. The connection between dynamics and canonical structure is established. Applications to theories with a gauge and constraints are of special interest. Several physical examples are given.

0. Introduction

Recent development in the theory of geometrical quantization (cf. [7, 11, 13]) has caused a growth of interest in the canonical structure of classical theories. There does not exist however up to now a general canonical formulation of classical field theory. Excepting few simple cases (e.g. the scalar theory $(\Box + m^2)\phi = G(\phi)$, cf. (1, 14]) it is not clear which physical quantities are to be taken as canonical variables and how to define Poisson brackets. Especially difficult are theories with a gauge. It seems that the best way to achieve good results in more complicated cases is "to make a lucky hit" of Poisson brackets. For theories in flat Minkowski space-time the Lorentz invariance is an important guide (cf. [3]). We think, however, that the existence of such fundamental structure as Poisson bracket in a given field theory can not depend on the question if 10 pc away the space-time is curved or not.

In the present paper we are going to formulate a general scheme of the canonical formalism which is consistent with all particular theories known to us. The starting point of our considerations is the finite-dimensional canonical formalism (theory of multisymplectic manifolds) given by one of us [9] [1]. It appears that after a deep reformulation one can apply this approach to a large class of observables (physical quantities, dynamical variables) which contains all physically interesting examples.

[1] Similar results concerning a finite-dimensional approach to canonical formalism was recently presented by Goldschmidt and Sternberg [17].