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Hyperfunction Quantum Field Theory

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Abstract. The quantum field theory in terms of Fourier hyperfunctions is constructed. The test function space for hyperfunctions does not contain C^∞ functions with compact support. In spite of this defect the support concept of H-valued Fourier hyperfunctions allows to formulate the locality axiom for hyperfunction quantum field theory.

§ 1. Introduction

In the usual framework of axiomatic quantum field theory, founded by Wightman [1], one assumes fields to be operator-valued *tempered distributions*. For nonrenormalizable interactions, however, the fields seem no longer remain tempered [2]. Several attempts have been made to extend Wightman's axioms for the quantum field theory so as to include a wider class of fields [2–4]. On the other hand, the recent development of the Euclidean field theory reveals that the temperedness of fields shows some inconvenience on coming back to the relativistic quantum field theory [5].

From the mathematical point of view the extension of Wightman's axioms starts with replacing the test function space $\mathscr{S}$ of tempered fields, the Schwartz space of rapidly decreasing functions, by its suitable dense subspace. In carrying through this program the most obstructive will be the axiom concerning the localizability of fields. The test function spaces considered so far by several authors contain C^∞ functions with compact support in configuration and/or momentum spaces. Hence the localizability of the fields has been preserved quite naturally in some way or other.

In the present paper we wish to formulate the quantum field theory in terms of *Fourier hyperfunctions*. The space of Fourier hyperfunctions is the dual of the space of rapidly decreasing holomorphic functions [6]. One of the characteristics of the latter space is that it is topologically invariant under Fourier transformations as is the case for the space $\mathscr{S}$. But since our space contains *no* functions of compact support, we are not allowed to state the locality of the field in the usual sense. In order to remedy this difficulty one of the present authors (S.N.),