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Critical Indices for Dyson's Asymptotically-Hierarchical Models

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Abstract. It is known that the investigation of the critical point for models of the type of Dyson's hierarchical models is reduced to the solution of some non-linear integral equation. In our previous publication the Gaussian solution was investigated. Here we construct non-Gaussian solutions of the equation and find the expressions for critical indices connected with them. Our procedure permits us to construct meaningful ε-expansions.

§ 1. Introduction

Dyson's hierarchical models or their generalization – asymptotically-hierarchical models – (a.h.m.) are of great interest because the renormalization group method in the theory of critical points by K. Wilson [3] and M. Fisher [4] becomes rigorous for such models (see [2] and the papers by Jona-Lasinio [5] and Gallavotti-Knops [6]). The investigation of critical points for a.h.m. is reduced to the solution of the corresponding nonlinear integral equation, which can be considered as an equation for the fixed point of the corresponding renormalization group. In [2, 8] a case with the Gaussian solution was investigated. It was shown that the critical indices in that case are precisely the same as predicted by the Landau semiphenomenological theory of phase transitions of the second kind. However, the Gaussian solution is stable only when the potential of interaction decreases sufficiently slowly.

In this paper we construct non-Gaussian solutions of our main integral equation. These solutions appear as bifurcations branches from the Gaussian solutions. The total number of the branches is infinite but only one of them has the necessary properties of stability to appear in general as a limit distribution for normed mean spin at the critical temperature. In the second part of this paper we find the values for critical indices corresponding to this branch. They coincide with the values found in the general theory by Wilson [3].

From the formal point of view the non-Gaussian solutions can be represented by a series of the parameter ε where ε is the deviation of the given value of the parameter from its bifurcation value. These series are always asymptotic because they describe the functions with different asymptotics at infinity. The method we apply can be regarded as a procedure which permits to make these ε-series meaningful. Roughly speaking at a given ε the formal series gives a good