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Note

Renormalization Problem in Nonrenormalizable Massless Φ^4 Theory

K. Symanzik

Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52,
Federal Republic of Germany

Abstract. Nonrenormalizable massless Φ^4 theory is made finite by regularization via higher derivatives in the kinetic part of the Lagrangean. The theory is shown to remain finite in the infinite cutoff limit if certain integrals over functions of one variable, with computable Taylor expansion at the origin, are finite. The values of these integrals are the only unknowns in the double series in powers of g and $g^{2/\varepsilon}$ obtained for the Green's functions in massless $(\Phi^4)_{4+\varepsilon}$ with generic ε. For $\varepsilon=1$ and $\varepsilon=2$, these series reduce to double series in powers of g and $\ln g$. The problems of extension to $(\Phi^4)_{4+\varepsilon}$ with mass, of causality and unitarity, of the relation to the BPHZ formalism, and of the indeterminacy of the result are discussed.

0. Introduction

Φ^4 theory in more than four space-time dimension is not renormalizable. This means that there is no choice of bare parameters such that all ultraviolet (UV) divergences in the perturbation theoretical construction are cancelled. Equivalently, construction of the perturbation expansions by BPHZ [1] or Epstein-Glaser [2] methods introduces an infinite number of arbitrary constants, and any finite choice of these constants apparently corresponds to actually employing a Lagrangean that is not the Φ^4 one.

Introducing a (sufficiently strong) cutoff Λ, however, removes all these UV divergences identically in all parameters. Our aim is to analyze the mechanism of this removal for $\Lambda<\infty$, to find sufficient conditions for the cancellation involved to persist in the limit $\Lambda\to\infty$, and to obtain and discuss the $\Lambda=\infty$ result if these conditions are satisfied. For a certain class of regularizations, this program can be carried out.

We introduce the cutoff in terms of higher derivatives in the kinetic part of the Lagrangean as proposed by Pais and Uhlenbeck [3], which is here equivalent to the method of Pauli and Villars [4]. Specifically, we set

$$L=-\tfrac{1}{2}\Phi\Box(1+\Lambda^{-2}\Box)\Phi-(1/4!)g_B\Phi^4-\tfrac{1}{2}m_{B0}^2\Phi^2\,, \tag{0.1a}$$

where

$$m_{B0}^2=\Lambda^2\textstyle\sum_{k=1}^{\infty}a_k(\varepsilon)\,(g_B\Lambda^{\varepsilon})^k \tag{0.1b}$$