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The Decay of the Bethe-Salpeter Kernel in $P(\varphi)_2$ Quantum Field Models*

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Abstract. We extend methods of high temperature expansions to show that for even weakly coupled $P(\varphi)_2$ quantum field models the Bethe-Salpeter kernel has 4 particle decay. More precisely if K denotes the Euclidean Bethe-Salpeter kernel

$$|K(x_1, x_2, x_3, x_4)| \leqq O e^{-m_0(1-\varepsilon)d_2},$$

where $x=(x^0, x^1)$, $d_2 = 2|x_1^0 + x_2^0 - x_3^0 - x_4^0| + |x_1^0 - x_2^0| + |x_3^0 - x_4^0|$ and $\varepsilon(\lambda)\to 0$ as $\lambda \to 0$. Our methods apply to other r particle irreducible kernels.

Introduction

In this paper we estimate the decay of r-particle irreducible kernels ($r \leqq 3$) for weakly coupled $\lambda P(\varphi)_2$ quantum field models. To obtain our estimates we extend the techniques of Glimm, Jaffe, and the author [1] which are related to high temperature expansions in statistical mechanics. See also [2]. A separate paper with Zirilli will use the decay of the two particle irreducible Bethe-Salpeter kernel to investigate the energy momentum spectrum of even $\lambda P(\varphi)_2$ models. For weak coupling we shall establish discreteness of the mass spectrum below $2m$ and (for $\lambda\varphi^4$) asymptotic completeness for states of mass less that $4m-\varepsilon$. Here m is the mass gap and $\varepsilon \to 0$ as $\lambda \to 0$. The detailed decay estimates of [3] also yield important information about the energy momentum spectrum such as the existence of single particle states. However such estimates do not seem to be formulated to give sufficient decay of the Bethe-Salpeter kernel. In statistical mechanics Minlos and Sinai [4] have made a detailed investigation of the spectral structure of the transfer matrix for Ising type models. Their techniques are vaguely related to ours.

The free Gaussian measure for the Euclidean field $\Phi(x)$ is denoted by $d\Phi(C) = d\Phi$ where the covariance is $C = (-\Delta + m_0^2)^{-1}$. Here Δ is the two dimensional Laplacian and m_0 is the bare mass. The action $V(\Lambda)$ in a region $\Lambda \subset R^2$ is defined by

$$V(\Lambda) = \lambda \int_\Lambda : P(\Phi(x)) : dx, \qquad x = (x^0, x^1), \tag{1.1}$$

where P is a positive polynomial. The Wick order is always defined with respect to $d\Phi$. The spatially cutoff expectation

$$\langle Q \rangle_\Lambda = \frac{\int e^{-V(\Lambda)} Q d\Phi}{\int e^{-V(\Lambda)} d\Phi} \tag{1.2}$$

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