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Stability in Linear Response and Clustering Properties

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Abstract. We derive a necessary and sufficient asymptotic condition assuring that a quantum dynamical system in equilibrium is stable in linear response.

We prove, in particular, that if the Hamiltonian has no singular-continuous spectrum and zero is the only eigenvalue, the dynamical system is stable.

Finally we prove that a dynamical system is strongly clustering, if and only if, it is weakly clustering and stable in linear response.

Introduction

In a previous paper [1] we considered an operator representation in a Hilbert space, $\tilde{\mathscr{H}}$, for the response, relaxation and correlation functions for any vector state, ω_0, of a von Neumann algebra $\mathfrak{M}$, acting on a Hilbert space $\mathscr{H}$, satisfying the K.M.S. condition, and we proved the existence of the static admittance, and the relaxation. In this paper we apply that technique to the study of the clustering properties of a dynamical system, indicating the usefulness of linear response theory.

Following the ideas of [2] we introduce (Definitions 1 and 2) the notion of stability of a dynamical system under a perturbation of the Hamiltonian, H_0, by a potential of the type λV, where V is any selfadjoint element of $\mathfrak{M}$, and λ, a real number, is the coupling constant. We derive (Theorem 3) an asymptotic condition which is necessary and sufficient to have stability in linear response. We prove, in particular, that if the Hamiltonian has no singular continuous spectrum and zero is the only eigenvalue, the dynamical system is stable in linear response (Theorem 4). Finally we give in Theorem 6 a necessary and sufficient condition for a dynamical system to be strongly clustering, namely: a dynamical system is strongly clustering if and only if it is weakly clustering, and stable in linear response.

Another relation between temporal cluster-properties and dynamical stability for pure thermodynamic phases can be found in Ref. [3].

I. Stability in Linear Response Theory

Let us consider an infinite quantum dynamical system in equilibrium, described by a von Neumann algebra, $\mathfrak{M}$, of observables, acting on a Hilbert space, $\mathscr{H}$, and a vectorial state, ω_0:

$$\omega_0(x) = (\Omega, x\Omega)\,,\; x \in \mathfrak{M}\,,$$

with Ω a cyclic element of $\mathscr{H}$.

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