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The Power Counting Theorem for Feynman Integrals with Massless Propagators

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Abstract. Dyson's power counting theorem is extended to the case where some of the mass parameters vanish. Weinberg's ultraviolet convergence conditions are supplemented by infrared convergence conditions which combined are sufficient for the convergence of Feynman integrals.

1. Introduction

In the theory of renormalization Dyson's power counting theorem plays a decisive part [1–3]. The contribution of a proper Feynman diagram to a Green's function has the form

$$J=\int dk R(k,p)$$

$$R=\frac{P}{\prod_{j=1}^{n}(l_j^2-m_j^2+i\varepsilon(\vec{l}_j^2+m_j^2))^{n_j}} \tag{1.1}$$

where

$$\begin{aligned} &k=(k_1\dots k_m)\,, && p=(p_1\dots p_N)\,,\\ &k_j=(k_{j0}k_{j1}k_{j2}k_{j3}), && p_j=(p_{j0}p_{j1}p_{j2}p_{j3})\,,\\ &dk=dk_1\dots dk_m\,, && dk_j\dots dk_{j0}dk_{j1}dk_{j2}dk_{j3}\,,\\ &m_j\geqq 0, \quad n_j>0\,. \end{aligned} \tag{1.2}$$

k_j and p_j are Minkowski vectors with the metric $(+1,-1,-1,-1)$. The vectors l_j are linear combinations

$$l_j=K_j(k)+P_j(p) \tag{1.3}$$

of the vectors $k_1,\dots,k_m$ and $p_1,\dots,p_N$ with $K_j\neq 0$. P is a polynomial in the components of k and p. The denominator of R is the common denominator of the unrenormalized integrand and the subtraction terms.

If all masses are non-zero Weinberg's version of the power counting theorem can be used to prove that the integral (1.1) is absolutely convergent provided the renormalized integrand R has been constructed according to Bogoliubov's subtraction rules [3, 4]. It can further be shown that the limit $\varepsilon\to+0$ exists as a covariant tempered distribution.

So far the power counting theorem has only been stated for non-vanishing masses. In the present paper Weinberg's ultraviolet convergence conditions are