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Existence of Phase Transition for a Lattice Model with a Repulsive Hard Core and an Attractive Short Range Interaction

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Abstract. We consider a lattice model with a repulsive hard core and an attractive short range interaction. We show that this model has at least three independent equilibrium states, when the temperature is sufficiently low and the chemical potential is suitably chosen.

1. Introduction

The existence of a phase transition at sufficiently low temperature and/or high density has been proved for a large class of lattice models [1].

The procedure, now actually standard, goes back to Peierls [2] and relies in showing that the correlation functions are sensitive to boundary conditions even in the thermodynamic limit.

The fundamental steps are:

i) definition of contours: each configuration on a lattice is associated to a family of polygonals (the contours);

ii) estimate of an upper bound for the probability of finding a given contour present: this probability, in all cases of interest, turns out to decrease esponentially with the inverse temperature and the lenght of the contour.

The main difficulty generally arises in getting point ii) and, until recently, it was possible to get these sort of estimates only for systems exhibiting a symmetry.

A big progress has been recently achieved by Pirogov and Sinai [3], that were able to prove point ii) for an Ising spin system with small non symmetric perturbations[1].

In the present paper we consider a lattice model with a repulsive hard core and an attractive two-body short range interaction and, by the method of Peragov and Sinai, we show that there are at least three equilibrium states corresponding to different boundary conditions if the temperature is sufficiently low and the chemical potential is suitably chosen.

[1] The case where three-body interactions only are present, was solved by Merlini, Hintermann, and Gruber [4], in two dimensions with a different technique, based on duality transformations.