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On Uniqueness of KMS States of One-dimensional Quantum Lattice Systems

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Abstract. Uniqueness of KMS states is proved for one-dimensional quantum lattice system. Sakai's theorem on uniqueness of KMS states is generalized to cases of non-commutative generators.

§ 1. Introduction

Uniqueness of equilibrium states for one-dimensional lattice system has been proved by Ruelle [7] for classical interactions and by Araki [1] for quantum interactions with a finite-range interaction. Simpler proofs have since been given for these cases (for example, see [8]. Also see Theorem 2 in [5]). It amounts to showing that any two states φ_1 and φ_2 satisfying the KMS condition are majorized by each other: $\varphi_1 \leqq \lambda\varphi_2 \leqq \lambda^2\varphi_1$ for some $\lambda > 0$.

We present here a proof of the uniqueness for one-dimensional quantum lattice system with an interaction Φ, which satisfies the same type of condition as known classical cases, namely surface energy has a bound independent of the volume. The key argument in the proof is Lemma 2 which states roughly that if the relative entropy of a state φ_1 with respect to a state φ_2 is finite, then the associated representation π_1 quasi-contains π_2.

To state the result more precisely, we use the following notation: The C^*-algebra $\mathfrak{A}$ under investigation will have the following structure as usual: For each integer v, $\mathfrak{A}$ has a subalgebra $\mathfrak{A}_v$ mutually commuting for different v. For any subset I of the set Z of all integers, $\mathfrak{A}(I)$ denotes the C^*-subalgebra of $\mathfrak{A}$ generated by $\mathfrak{A}_v$, $v \in I$. We assume that each $\mathfrak{A}_v$ is a type I finite factor and $\mathfrak{A}(Z) = \mathfrak{A}$. For each finite subset Λ of Z, an interaction potential $\Phi(\Lambda) \in \mathfrak{A}(\Lambda)$ is given such that

(0) $\Phi(\emptyset) = 0$,

(1) $\|\Phi\|_\alpha \equiv \sup_v \sum_\Lambda \{e^{\alpha N(\Lambda)} \|\Phi(\Lambda)\| ; v \in \Lambda\} < \infty$,

where $N(\Lambda)$ denotes the number of points in Λ and $\alpha > 0$,

(2) the following element $W(\Lambda_n)$ of $\mathfrak{A}$ for an increasing sequence of finite subsets Λ_n of Z is bounded in norm uniformly in n:

$$W(\Lambda) \equiv \sum_J \{\Phi(J); J \subset\subset Z, J \cap \Lambda \neq \emptyset, J \cap \Lambda^c \neq \emptyset\} . \tag{1.1}$$

Here Λ^c denotes the complement of Λ in Z and $\subset\subset$ denotes a finite subset.

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