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Integral Representations for Schwinger Functionals and the Moment Problem over Nuclear Spaces

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Abstract. It is shown that a continuous positive linear functional on a commutative nuclear *-algebra has an integral decomposition into characters if and only if the functional is strongly positive, i.e. positive on all positive polynomials. When applied to the symmetric tensor algebra over a nuclear test function space this gives a necessary and sufficient condition for the Schwinger functions of Euclidean quantum field theory to be the moments of a continuous cylinder measure on the dual space. Another application is to the problem of decomposing a Wightman functional into states having the cluster property.

1. Introduction

Whereas the extremal states of an abelian *-algebra of bounded operators on Hilbert space are at the same time the characters of the algebra, this is no longer true for algebras of unbounded operators[1]. In a previous article [2] an integral decomposition theory associated with the weak commutant of families of unbounded operators was used to obtain an extremal decomposition of states on nuclear *-algebras. The present paper is concerned with decompositions into characters in the commutative case. It is shown that such a decomposition is possible if and only if the state, satisfies a positivity condition which is well known from the classical moment problem over finite dimensional spaces [3, 4]. This result can be applied to Euclidean quantum field theory where the sequence of Schwinger distributions defines by assumption a positive linear functional on the symmetric tensor algebra over some nuclear space of test functions. The condition tells also when a Wightman functional is an integral over states having the cluster property. That this is not always the case was shown in [2].

The infinite dimensional moment problem has been treated by several authors under conditions which at the same time guarantee the uniqueness of the solution, cf. e.g. [5] and [1]. Our method is based on the extension theory in [2] which, however, has to be modified slightly to fit our purpose. These changes are fairly straightforward so we can in most cases refer to [2] for the proofs. This method, which might appear somewhat indirect if one is only aiming at a solution of the moment problem (i.e. our Theorem 4.3)[2], has some advantages: It makes explicit the intimate connection of the solution with the weak commutant of the operators

[1] See e.g. [1], Theorem 5.5.

[2] After this research was completed a more direct proof of Theorem 4.3 was found by G. C. Hegerfeldt. A closely related result has also been obtained by M. Dubois-Violette (private communication).