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The $:\phi^2:$ Field in the $P(\phi)_2$ Model

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Abstract. Euclidean Field Theory techniques are used to study the Schwinger functions and characteristic function of the $:\phi^2:$ field in even $P(\phi)_2$ models. The infinite volume limit is obtained for Half-Dirichlet boundary conditions by means of correlation inequalities. Analytic continuation yields Lorentz invariant Wightman functions. It is shown that, in the infinite volume limit, $\langle :\phi(x)^2:\rangle \geqq 0$ for both the Half and the Full-Dirichlet $(\lambda\phi^4)_2$ model. This result also holds for a finite volume with periodic boundary conditions.

1. Introduction

The path space approach to the self-interacting scalar Bose field in two space-time dimensions involves the introduction of the free Euclidean field ϕ which can be viewed (Nelson [1]) as the generalized Gaussian stochastic process $\phi(x)$ with mean zero and covariance

$$S(x-y) = \langle \phi(x)\,\phi(y)\rangle = \int \frac{d^2(p)}{(2\pi)^2}\,\frac{e^{ip\cdot x}}{p^2+m^2}\,. \tag{1}$$

The Schwinger functions associated with the $P(\phi)$ interaction in the open bounded region $\Lambda \subset R^2$ are given by

$$\langle \phi(f_1)\ldots\phi(f_n)\rangle_\Lambda = \frac{\int \phi(f_1)\ldots\phi(f_n)\, e^{-\int_\Lambda :P(\phi(x)):\,d^2(x)}\, d\mu_0}{\int e^{-\int_\Lambda :P(\phi(x)):\,d^2(x)}\, d\mu_0}\,, \tag{2}$$

where $\operatorname{supp} f_i \subset \Lambda$, $\phi(f) = \int d^2(x)\,\phi(x)\,f(x)$, and μ_0 is the free Gaussian measure, i.e. the Gaussian measure associated with the free Euclidean field ϕ. One is then interested in the Schwinger functions in the infinite volume limit $\Lambda \to R^2$. In the case of small coupling constant, the Glimm-Jaffe-Spencer [2] cluster expansion is a powerful tool for studying this infinite volume limit. If, however, it is desired to obtain results independent of the magnitude of the coupling constant, then correlation inequalities of Griffiths' type become the primary tool (Guerra, Rosen, and Simon [3], see also Simon [4]). This method is essentially restricted to even polynomial interactions and we assume this in the following.

Little is known about correlation inequalities for Wick powers (see [3]; for small coupling Wick powers have been studied by Schrader [12]) so it is of some interest to explore the properties of even the simplest Wick power, $:\phi^2:$. Indeed,