Commun. math. Phys. 43, 73—87 (1975)

On Local Field Products in Special Wightman Theories*

Klaus Baumann

Bell Laboratories, Murray Hill, New Jersey, USA, and Max-Planck-Institut für Physik und Astrophysik, München, Federal Republic of Germany

Received January 23, 1975

Abstract. We shall try to define local field products under assumptions imposed only on the four-point-function. This idea is based on the work of Schlieder and Seiler [1].

In our framework we shall prove that the two-point-function carries the strongest singularity whenever two arguments in a Wightman function coincide. This will be generalized to the case when more arguments coincide. We shall define "regulated" n-point-functions and study their properties in detail. This will lead us to the definition of arbitrarily high powers of the field-operators as operator-valued distributions over $\mathscr{D}(\mathbb{R}^4)$ in the center coordinate with a dense domain of definition.

1. Introduction and Some Results Stated in [1]

Field products at the same space-time point lead to great difficulties in quantum theories because of the distributional character of the field operators.

Schlieder and Seiler [1] define local products of two field operators under assumptions imposed only on the four-point-function. We want to extend their approach such that it includes local products of three or more field operators. Our investigation is based on axiomatic quantum field theory [2] described in terms of Wightman functions.

Let us first introduce some notations:

$$\underline{z} := (z_0, \ldots, z_n) \in \mathbb{C}^{4(n+1)}$$

$$\underline{\zeta} := (\zeta_1, \ldots, \zeta_n) \in \mathbb{C}^{4n} \quad \text{with} \quad \zeta_i = z_i - z_{i-1}$$

$$\tau_n^{\pm} := \{\underline{\zeta} \in \mathbb{C}^{4n} \mid \operatorname{Im} \zeta_i \in V_n^{\pm}\}$$

("forward/backward tube")

$$\tau_n' := \{\underline{\zeta} \in \mathbb{C}^{4n} \mid \exists \Lambda \in L_+(\mathbb{C}) : \Lambda \underline{\zeta} \in \tau_n^+\}$$

("extended tube")

where $L_+(\mathbb{C})$ denotes the proper complex Lorentz group. For $\pi \in S_{n+1}$ (group of permutations of $\{0, 1, \ldots, n\}$) we define

$$\underline{\zeta}_\pi := (z_{\pi(1)} - z_{\pi(0)}, \ldots, z_{\pi(n)} - z_{\pi(n-1)}) = \left(\sum_{j=1}^{\pi(1)} \zeta_j - \sum_{j=1}^{\pi(0)} \zeta_j, \ldots, \sum_{j=1}^{\pi(n)} \zeta_j - \sum_{j=1}^{\pi(n-1)} \zeta_j \right)$$

$$\tau_{n,\pi}^{\pm,\prime} := \{\underline{\zeta} \in \mathbb{C}^{4n} \mid \underline{\zeta}_\pi \in \tau_n^{\pm,\prime}\}$$

("permuted forward/backward/extended tube").

* Based in part on the author's thesis, University of Munich, Germany, October 1974.