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Sommerfeld-Watson Representation for Double-Spectral Functions

II. Crossing Symmetric Pion-Pion Scattering Amplitude without Regge Poles

J. S. Frederiksen

Research School of Physical Sciences, The Australian National University, Canberra, Australia 2600

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Abstract. We discuss, for the case of pion-pion scattering, a closed system of equations which may be used for a self-consistent calculation of partial-wave amplitudes. It is shown that, for a given sufficiently small input function, the equations have a locally unique solution in a particular Banach space of doubly Hölder continuous partial wave amplitudes. At a fixed point, the scattering amplitude is shown to satisfy both a crossing symmetric unsubtracted Mandelstam representation and the elastic unitarity condition. In this initial study the partial-wave amplitudes are holomorphic in the right half complex angular-momentum plane.

1. Introduction

Since Mandelstam [1] in 1958, proposed the double spectral representation for the two-particle scattering amplitude there have been numerous attempts to solve the crossing-unitarity equations [2]. While dispersion theory has led to some fruitful phenomenological correlations of experimental data, these attempts have on the whole met with limited success.

Several years ago, Atkinson [3] proposed a list of remedies for the then diseased state of strong-interaction dynamics; parts of this programme were subsequently implemented [3–5]. In particular, it was shown by means of fixed-point theorems that there exist crossing symmetric pion-pion scattering amplitudes satisfying the Mandelstam representation with elastic unitarity between the elastic and inelastic thresholds and the inelastic unitarity inequality above the latter. However, if more than one subtraction is needed in the double dispersion relation for the amplitude, it has not been possible to guarantee positivity and boundedness of the partial-wave amplitude $A(s, l)$ as $s \to \infty$. Now it is known that the f-meson resonance, with spin 2 occurs in the pion-pion spectrum and thus, at least three subtractions are expected in the dispersion relation.

The difficulty with the iteration equations for the double spectral function lies in the fact that, for the partial-waves to be bounded as $s \to \infty$, the double spectral function must have infinite oscillations. However, it is very difficult to incorporate suitable oscillations in the Banach space of double spectral functions. It was proposed in a previous paper [6] (referred to as I) that a likely way of overcoming this problem is to replace the iteration equations for the double spectral function by a closed system of equations for $A(s, l)$, the continuation of