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Higher Order Estimates for the Yukawa$_2$ Quantum Field Theory*

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Abstract. Higher order estimates of the form

$$\prod_1^n N_{\tau_i} \leqq \text{const}(H(g)+I)^n, \qquad \sum_1^n \tau_i < 1, \tau_i \geqq 0$$

are proved for the Yukawa$_2$ models with and without SU$_3$ symmetry. We also prove norm convergence of $\Pi_1^n N_{\tau_i}^{\frac{1}{2}} \cdot R_\kappa^{n/2+\delta}$ as $\kappa \to \infty$ where $R_\kappa = (H(g,\kappa)+I)^{-1}$.

Introduction and Results

Higher order estimates, bounding powers of the fractional energy operator by powers of the Hamiltonian, have proved useful in studying the $\mathscr{P}(\phi)_2$ model [1]. In this paper we obtain similar estimates for the Yukawa$_2$ model as well as for the Yukawa$_2$ model with internal SU$_3$ symmetry discussed in [2].

In the following we will use even, positive odd and negative odd values of ε to label bosons, fermions and anti-fermions respectively. Thus $b(k,\varepsilon)$ denotes the annihilation operator for free particles of momentum k and type ε. The fractional energy operator is:

$$N_\tau = \sum_\varepsilon N_\tau^{(\varepsilon)} = \sum_\varepsilon \int dk\, \mu(k,\varepsilon)^\tau\, b^*(k,\varepsilon)\, b(k,\varepsilon)\,,$$
$$\mu(k,\varepsilon) = (k^2 + m(\varepsilon)^2)^{\frac{1}{2}}\,,$$

where $m(\varepsilon)=m$ for bosons, $m(\varepsilon)=M$ for fermions. For convenience we define $E(k)=\sqrt{k^2+1}$. We will work with the dense domain $\mathscr{D}$ of vectors in Fock space with finite numbers of particles and wave functions in Schwartz space.

Formally, the finite volume Hamiltonian $H(g)$ has the form

$$\begin{aligned} H(g) &= H_0 + H_I(g) + C(g) \\ &= N_1 + \lambda \int dx g(x) : \overline{\psi}\psi\phi : - \tfrac{1}{2}\delta m^2 \int dx g^2(x) : \phi^2 : - E(g)\,, \end{aligned}$$

where $g \geqq 0 \in C_0^\infty$ and δm^2, $E(g)$ provide infinite renormalizations. To define the momentum cutoff Hamiltonian $H(g,\kappa)$ we multiply the momentum space kernels $w^c(k,p_1,p_2)$, $w(k,p_1,p_2)$ of the interaction term $H_I(g)$ by a general momentum cutoff function $\chi_\kappa(k,p_1,p_2)$ in the sense of [3]. The renormalization constants

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