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Note

Limit Theorems for Multidimensional Markov Processes

G. Gallavotti*

Instituut voor Theoretische Fysica, Katholieke Universiteit Nijmegen, Nijmegen, The Netherlands

G. Jona-Lasinio

Istituto di Fisica dell'Università, Gruppo GNSM, Padova, Italy

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Abstract. An informal exposition of some recent results and conjectures.

A multidimensional Markov process (mdmp) is a dynamical system (K, m, T) where:

K space of the sequences of symbols from a finite alphabet $I = (a, b, \ldots z)$ indexed by the elements $\eta \in Z^d \equiv$ lattice formed by the d-ples of integers. K is regarded as $K = \Pi_{\eta \in Z^d} I$ i.e. as a product space of copies of I; furthermore I is topologized by the discrete topology and K by the product topology.

T is the translation group acting, in the natural way, on K: if $\underline{\sigma} \in K$, $\underline{\sigma} = \{\sigma_\xi\}_{\xi \in Z^d}$ then $T_\eta \underline{\sigma} = \underline{\sigma}' = \{\sigma_{\xi+\eta}\}_{\xi \in Z^d}$, if $\eta \in Z^d$.

m is a regular complete probability measure on K whose σ-field contains all the open sets of K. Furthermore m has the "Markov property".

The Markov property can be easily expressed as a requirement on the conditional distributions associated with finite sets $\Lambda \subset Z^d$. Let $\underline{\sigma}_\Lambda = \{\sigma_\xi\}_{\xi \in \Lambda}$ $\underline{\sigma}' = \{\sigma_\xi\}_{\xi \in Z^d \setminus \Lambda}$; then, with obvious notations, $\underline{\sigma}_\Lambda \cup \underline{\sigma}' \in K$ and we can define $m_\Lambda(\underline{\sigma}_\Lambda / \underline{\sigma}')$ as the conditional probability that a configuration $\underline{\sigma} \in K$ coincides with $\underline{\sigma}_\Lambda$ inside Λ once it is known that, outside Λ, $\underline{\sigma}$, and $\underline{\sigma}'$ coincide. The Markov property is then the following [5, 17]:

mp for m-almost all $\underline{\sigma}_\Lambda \cup \underline{\sigma}'$ in K the functions $m_\Lambda(\underline{\sigma}_\Lambda / \underline{\sigma}')$ depend on $\underline{\sigma}'$ only through the values σ'_ξ with $\xi \in \partial\Lambda \equiv$ {set of lattice points not in Λ but located at unit distance from Λ}. Here Λ is an arbitrary finite subset of Z^d. Furthermore, $m_\Lambda(\underline{\sigma}_\Lambda / \underline{\sigma}') > 0$ m-a.e. $\forall \Lambda \subset Z^d$.

In the following we shall assume, for simplicity, that I is a two symbol alphabet $I = \{-1, +1\}$.

The following very interesting structure (and existence) theorem for mdmp holds: [5, 10, 15, 17].

Theorem. *All ergodic mdmp in d-dimensions can be obtained as follows:*

i) *choose* $d+1$ *real numbers* $\beta_1, \ldots, \beta_d, h$;

ii) *choose* $\underline{\sigma}^0 \in K$;

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* On leave of absence from Istituto di Fisica Teorica, Università di Napoli, Napoli, Italia.