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Absence of Ordering in a Class of Lattice Systems

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Abstract. A generalized form of the classical Bogoliubov inequality obtained by Mermin is derived for all lattice systems whose configuration manifold is a compact connected real Lie group G; the new inequality relates elements of $C^{\infty}(G;\mathbb{C})$, the algebra of indefinitely differentiable complex-valued functions on G. We use it to prove the absence of ordering in a class of one- and two-dimensional systems defined by G-invariant Hamiltonians. This class contains in particular the Stanley model for ferromagnets and a lattice version of the Maier-Saupe model for nematic liquid crystals.

1. Introduction

It has been shown by Mermin and Wagner [1] that the isotropic Heisenberg model cannot exhibit a spontaneous magnetization at any finite temperature in one and two dimensions, provided the interactions have not too long a range. Their proof exploits an inequality due originally to Bogoliubov [2], which relates any linear self-adjoint operator, acting on some finite-dimensional Hilbert space, to two other linear operators on the same space by means of a thermal average [3].

More recently, Mermin [4] has derived an inequality of the same kind by purely classical arguments, in order to show the absence of spontaneous magnetization in various one- and two-dimensional classical spin systems; in all these cases, the inequality relates functions on a suitably chosen phase space, but is in fact only valid if certain contributions in the thermal averages vanish.

It is the object of this note to show that a generalized form of the classical Bogoliubov inequality obtained by Mermin can be derived for all lattice systems whose configuration manifold is a compact connected real Lie group G; the new inequality relates elements of $C^{\infty}(G;\mathbb{C})$, the algebra of indefinitely differentiable, complex-valued functions on G. We use it to rule out the existence of a non-zero "order parameter" at any finite temperature in a class of one- and two-dimensional systems defined by G-invariant Hamiltonians.

The proof of the generalized Bogoliubov inequality is given in Section 2; the main theorem concerning the absence of long-range order is proved in Section 3; Section 4 is concerned with an example which shows that the class of lattice systems considered contains most of the classical vector models generally used in the field of critical phenomena. This class includes in particular the Stanley model for ferromagnets [5] and a lattice version of the Maier-Saupe model for nematic liquid crystals [6, 7].