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Dynamical Stability and Pure Thermodynamic Phases

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Abstract. A notion of stability of dynamics under distant perturbations is introduced. It is demonstrated, for quasi-local systems, that the stability of an equilibrium state under the same perturbations implies the state is factorial, i.e. strongly clustering in space. We also characterize the set of perturbations necessary to ensure the equivalence of stability and factorialness.

1. Introduction

Several characterizations of pure thermodynamic phases have been proposed; each such proposal emphasizes a different aspect of the equilibrium phenomena. The aspects which have received most attention are either *kinematical*

a) spatial cluster properties,

b) extremality among the class of all equilibrium states,

or *dynamical*

c) stability under dynamical perturbations,

d) temporal cluster properties.

The purpose of this note is to examine some of the interrelationships between properties a, b, and c, in the framework of quantum statistical mechanics.

For simple quantum models, such as spin systems with short range interactions, the connections between points a and b, have been reasonably well understood. The dynamics in these models is provided by a strongly continuous one-parameter group of automorphisms τ of a C^* algebra $\mathfrak{A}$ of kinematical observables and the thermodynamic limits of finite volume Gibbs equilibrium states form the set (or possibly a subset) of τ-KMS states over $\mathfrak{A}$. These states, ω, form a convex set with extremal points; a state is extremal in this set if, and only if, it is a factor state. On the other hand a state of such a system is factorial if, and only if, it possesses a strong spatial cluster property of the Powers type. We demonstrate that the states which are stable under "distant" dynamical perturbations are factor states and hence provide a connection with property c.

The stability criterion is of the following nature. For each $P=P^*\in\mathfrak{A}$ we define local perturbations ω_P and τ^P of ω and τ and then consider sets of perturbations, typically perturbations which recede in configuration space, which leave the dynamics stable in the sense that:

$$\|\tau_t^{P_\alpha}(A)-\tau_t(A)\|\xrightarrow[\alpha]{}0\,,\qquad A\in\mathfrak{A}\,.$$