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A Characterization of Clustering States

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Abstract. A characterization of states, over quasi-local algebras, which satisfy a strong cluster property is derived. The discussion is applicable to classical systems and quantum systems with Bose or Fermi statistics.

1. Introduction

Several years ago Powers characterized primary states over UHF algebras as the states satisfying a certain singly uniform cluster property [1]. The importance of cluster properties in physical theories led several authors to generalize Powers results to other algebras encountered in field theory or statistical mechanics (see for example [2–6]). These generalizations took several different directions; the idea of far away observables, and analogy with Sinai's results on K-systems [7], is emphasized in [2, 3]; the notion of relative commutants of observables is used in [4]; the singly uniform clustering property is equated with a doubly uniform clustering property in [5, 6]. Most of these generalizations were, however, modeled on certain commutation properties which are typically encountered in classical mechanics or quantum mechanics with Bose statistics. The only attempt to characterize clustering states of Fermi systems occurs in [5] which considers only the even states of the CAR-algebra. Also the proof of the result concerning these states, Proposition 4.5 of [5], is incomplete[1].

The purpose of this note is to correct this situation by providing a general discussion which applies to all standard systems encountered in statistical mechanics regardless of statistics.

2. Quasi-local Systems

In this section we discuss the basic structure of algebras which are generated by local subalgebras and possibly satisfy Fermi statistics.

Throughout $\mathscr{F}$ will denote an index set with an order relation $\geqq$. We always assume that $\mathscr{F}$ is directed with respect to this relation, i.e. if $\alpha, \beta \in \mathscr{F}$ then there exists a $\gamma \in \mathscr{F}$ such that $\gamma \geqq \alpha, \beta$. In the second half of this section we also assume the existence of an orthogonality relation $\perp$ between pairs of elements of $\mathscr{F}$ with the following properties

a) if $\alpha \leqq \beta$ and $\beta \perp \gamma$ then $\alpha \perp \gamma$,

b) if $\alpha \perp \beta$ and $\alpha \perp \gamma$ there exists a $\delta \in \mathscr{F}$ such that $\alpha \perp \delta$ and $\delta \geqq \beta, \gamma$.

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[1] $\mathfrak{A}(M_{a_0})^-$ and $\mathfrak{A}(M_a)' \wedge \mathfrak{A}^-$ do not generate $\mathfrak{A}_a^-$ as is claimed in the proof given in [5].