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A Class of Inhomogeneous Cosmological Models

P. Szekeres

Department of Mathematical Physics, University of Adelaide, Adelaide, Australia

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Abstract. All solutions of Einstein's field equations representing irrotational dust and possessing a metric of the form $ds^2 = dt^2 - e^{2\alpha}dr^2 - e^{2\beta}(dy^2 + dz^2)$ are found. The new metrics generalize the earlier Bondi-Tolman, Eardley-Liang-Sachs, and Kantowski-Sachs cosmological models.

1. Introduction

Cosmological solutions to Einstein's field equations are generally found [1] by imposing a large group symmetries on the metric, amounting to an assumption of spatial homogeneity. While this assumption is probably quite reasonable in an averaged sense, it is obviously false on galactic and smaller scales. It is therefore of considerable interest to have at hand a large variety of inhomogeneous models as a basis of comparison with the homogeneous ones. All questions of detail such as galaxy formation or the detailed structure of the black-body radiation, to be treated properly, should ultimately be referred to such inhomogeneous models. In particular the study of the singularity structure of more general models is still only in its initial stages [2], and promises to lead to radical departures from the highly idealized Friedmann-type singularities which are usually considered. However the only generally known inhomogeneous solutions are the spherically symmetric Bondi-Tolman metrics [3] and the plane-symmetric models of Eardley-Liang and Sachs [2] (referred to as ELS in this paper), and both of these examples still impose strong symmetry groups.

In this paper we abandon all *a priori* symmetry assumptions, but concentrate our attention on metrics having the simple form

$$ds^2 = dt^2 - e^{2\alpha}dr^2 - e^{2\beta}(dy^2 + dz^2). \tag{1}$$

All solutions of this type will be found which represent irrotational dust,

$$G_{\mu\nu} = \kappa T_{\mu\nu} = \kappa\varrho u_\mu u_\nu \tag{2}$$

where $u_\mu = (1, 0, 0, 0)$, $\kappa = 8\pi G (c = 1)$. Our solutions will be found to generalize both the Bondi-Tolman and ELS solutions, by essentially displacing the "centres of symmetry" in a well-determined manner, leading to what might be termed quasi-spherical and quasi-planar metrics (as well as a new class of quasi-pseudo-spherical metrics which generalize a previously unexplored class of pseudo-spherically symmetric metrics). A second class of solutions is also found which generalizes the homogeneous cylindrical solutions of Kantowski-Sachs [4] in an analogous fashion. Wheter this displacement process is part of a more