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The Structure Theorem in S-Matrix Theory

D. Iagolnitzer

Service de Physique Théorique, Centre d'Études Nucléaires de Saclay, Gif-sur-Yvette, France

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Abstract. A basic tool in the derivation of multiparticle discontinuity formulae in S-matrix theory is a "structure theorem" which proves analyticity properties for integrals of products of scattering functions [1, 5, 7].

We present here some recent mathematical results and show how they provide directly a general form of this theorem. This new proof, which removes an unnecessary technical assumption of the previous ones, is a development of a method proposed by Pham [8].

I. Introduction

The basic quantities of interest in the relativistic quantum physics of systems of massive particles with short-range interactions are the scattering functionals S_{IJ} between sets I and J of initial and final particles. From general quantum principles, each S_{IJ}, or its "connected part" S^c_{IJ}, is known [1, 2] to be a tempered distribution, which is defined on the space of all real on-mass-shell initial and final energy-momentum 4-vectors $p_k(p_k^2 = p_{k0}^2 - \boldsymbol{p}_k^2 = m_k^2,\ (p_k)_0 > 0)$ and contains an energy-momentum conservation δ-function:

$$S^c_{IJ} = T_{IJ} \times \delta^{(4)}\left(\sum_{i \in I} p_i - \sum_{j \in J} p_j\right). \tag{1}$$

The distribution T_{IJ} is defined on the physical-region $\mathscr{M}_{IJ}$ of the process $I \to J$ (i.e. the set of all real 4-momenta p_k satisfying the above mentioned mass-shell constraints and the further condition $\Sigma p_i = \Sigma p_j$).

Decisive advances have been made at the end of the sixties in the general derivation and understanding of the physical-region analytic structure of the distributions T_{IJ}. On the one hand, a macroscopic causality property has been stated and proved to be equivalent to some basic analytic properties of T_{IJ} [3, 4]. These properties ensure in particular that for each process $I \to J$, there is a unique analytic function F_{IJ} (defined in a domain of the complexified mass-shell $\mathscr{M}^c_{IJ}$) to which T_{IJ} is equal at all points which do not lie on $+\alpha$-Landau surfaces of connected graphs, and from which it is a "plus $i\varepsilon$" boundary value at almost all $+\alpha$-Landau points.

On the other hand, a general form has been derived from unitarity for the discontinuities of the scattering functions around the $+\alpha$-Landau singularities [5, 6]. The usefulness of this result in various contexts is described elsewhere (see for instance [1, 2] and the original references quoted therein). Its derivation