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Correlation Inequalities for Ising Spin Lattices*

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Abstract. A change of spin representation is used to present expectation inequalities on Ising lattices directly as sums of terms of like sign. The technique is extended to correlation inequalities by introducing replica variables which convert correlations into expectations on a larger space. Second order correlations are analyzed in full from this viewpoint, recovering the FKG set, among others. Third order correlations are examined in some detail, and the sign of the multi-site Ursell correlations F_3, F_4, F_6 established under appropriate restrictions.

1. Introduction

Problems of substance in mathematical physics can rarely be solved exactly. Indeed, a large part of the field is devoted to developing approximation methods for dealing with otherwise intractable situations. However, the reliability of such methods is always suspect unless the magnitude of the error committed can at least be estimated. Still better, one can in increasingly many cases establish bounds

$$A_1 \leqq A \leqq A_2 \tag{1.1}$$

for the value of a desired quantity A. Since there is usually no inate applicable symmetry, very different methods may have to be used to establish upper and lower bounds. But, once established, such bounds can often be pyramided to bounding other quantities [1], comparing characteristics of similar systems [2], and the like.

It is the purpose of this paper to set up a suitable apparatus for bounding local observables in Ising spin lattices. We first review the relevant material concerning free energies, expectations, and correlations, then show how a change of representation can be an effective tool for establishing expectation inequalities. The replica variable formalism is introduced to extend this technique to correlations, and the problem of second order correlations fully analyzed within the bounds of the method. Finally, higher order Ursell correlations are introduced, and several are analyzed as part of the general investigation of higher order correlations.

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