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Descending Problem in Green's Function Approach to Quantum Field Theory

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Abstract. The question how to determine lower many-point functions in terms of higher ones, which we call the descending problem, is discussed for the $(\phi^4)_{1+3}$ model of quantum field theory. Equations to be considered are non-linear non-compact operator equations in complex Banach spaces.

Several sufficient sets of conditions for convergence of successive approximation schemes are presented for small values of the renormalised coupling constant. Local uniqueness of solution is proved under certain conditions.

I. Introduction

Usually, quantum field theory is concerned with expression of higher many point functions in terms of lower ones. But as will be discussed in Chapter VI, this problem does not seem to have unique solutions even for polynomially non-linear interactions unless a perturbative approach makes sense. Now let us ask the converse question. Suppose G_N (N fixed) were known or substituted by a model function satisfying the causality condition etc. Is it then possible to determine G_n $(n<N)$? This is a relevant question, because 1) for example, for Yukawa type interaction the lowest observable processes correspond to four point functions; and 2) if one begins with G^0 (bare propagator) and point vertex, one ends up with divergences and ghosts except in superrenormalisable models. (Here G_2 stands for the two point function in the Heisenberg representation and $G_n (n \geqq 4)$ stand for amputated connected n-point functions.)

In this note we pursue the Green's function approach to quantum field theory. By Green's function approach we mean that once the equations for Green's functions (many-point functions) have been derived, one can forget field operators and deal exclusively with Green's functions.

Let us take the $(\phi^4)_{1+3}$ model and suppose that G_4 is given. Then our problem is concerned with the existence and uniqueness of the following equation:

$$\begin{aligned} &G_2(p)-(p^2-m_r^2-i\varepsilon)^{-1}\, g_r \int_{m_r^2}^{p^2} d(p'^2) \int_{m_r^2}^{p'^2} d(p''^2) \frac{d^2}{d(p''^2)^2} \\ &\quad\cdot \int d^4 q_1\, d^4 q_2\, G_2(q_1)\, G_2(q_2)\, G_2(p''-q_1-q_2)\, G_4(q_1, q_2, p''-q_1-q_2, -p'') \qquad (1.1)\\ &\quad\cdot G_2(p)=(p^2-m_r^2-i\varepsilon)^{-1} \equiv G^0(p)\,. \end{aligned}$$

This equation incorporates a technique of renormalisation introduced by Taylor [1]. Let us define σ as follows

$$\sigma(p^2)=G_2(p)-G^0(p)\,.$$