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Mean Square Relaxation Times for Evolution of Random Fields

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Abstract. We consider the problem of relaxation times for Markov evolution of systems composed of a countable number of locally interacting particles, each one of which has a finite phase space. We give a theorem for comparison of mean square relaxation times of evolutions possessing the same ergodic stationary state. We give a reduction theorem for "attractive" evolutions. The results are applied to a generalization of the Glauber evolution of the one dimensional Ising chain.

1. Introduction

This paper concerns the stochastic time evolution of a system composed of a countable number of locally interacting particles, each one of which possessing only a finite number of states. A basic example of this type of system is the Glauber model [5] for the evolution of the one dimensional Ising chain.

The joint configuration of the particles is described by a point in the phase space Ω. The equilibrium distribution is a probability measure μ on Ω. We use the operator T_t to express the evolution of the function f on Ω. The time development is specified by the master equation

$$\frac{d}{dt}(T_t f) = G(T_t f) \tag{1.1}$$

with a given master operator G.

The properties of the equilibrium state μ are usually well known. Very much less is known about the exact nature of the master operator. One fundamental constraint on G is that it should have μ as a fixed point. A second commonly imposed constraint is that G should satisfy the "principle of detailed balance". In probabilistic parlance this amounts to reversibility of G with respect to μ. The physical interpretation is that fluctuations from equilibrium should not distinguish the direction of time.

The particular concern of this paper is the convergence properties of solutions to (1.1) in the limit as t approaches infinity. Assume there is a function space $\mathscr{F}$ which is complete in a certain norm $\|\cdot\|$ such that for each $f \in \mathscr{F}$ there is an $f_\infty \in \mathscr{F}$ with

$$\lim_{t\to\infty} e^{\lambda t} \| T_t f - f_\infty \| = 0 \tag{1.2}$$

for some fixed $\lambda \geqq 0$. Then λ is called a *relaxation coefficient for T_t in the norm* $\|\cdot\|$. The *relaxation time of T_t in the norm* $\|\cdot\|$ is the reciprocal of the supremum of the set of relaxation coefficients. If there is a strictly positive relaxation coefficient,