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Phase Transitions in Classical Heisenberg Ferromagnets with Arbitrary Parameter of Anisotropy

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Abstract. The existence of a phase transition of the first kind is proved for anisotropic classical Heisenberg ferromagnet in two or more dimensions and with arbitrary parameter of anisotropy α, $|\alpha| < 1$; a similar fact is proved for much more general lattice spin systems.

Introduction

Bortz and Griffiths proved lately (see [1]) the existence of a phase transition for sufficiently low temperatures in anisotropic classical Heisenberg ferromagnets with small parameter of anisotropy $\alpha(|\alpha| < 0.0298$ and $|\alpha| < 0.0198$ for a square lattice and simple cubic lattice, respectively). Here we prove the similar result for any α, $|\alpha| < 1$. It is the known Fisher's hypothesis (see [2]).

Theorem 1 of our paper contains essentially more general conditions for the existence of a phase transition of the first kind in lattice spin systems with continuous spin space.

The main difference between our method and the method of [1] is the following: in [1] the sharp "border" is constructed and we construct a spread gradually altering "border" (Bloch wall).

It is interesting to compare our result with the result of Mermin and Wagner (see [3]) about the impossibility of phase transitions of the sort considered here for the square lattice and for $|\alpha| = 1$.

1. Formulation of the Main Result

Let $\mathbb{T}$ be an abelian group $\mathbb{Z}^\nu$, $\nu \geqq 2$, where $\mathbb{Z}$ is a group of integers. Let S be a compact separable metric space with finite nonnegative measure μ defined on Borel subsets of S. Assume to be given a real measurable function $U(s_1, s_2) = U(s_2, s_1)$ on $S \times S$ which is bounded from below on $S \times S$.

We shall consider Gibbsian random fields on a lattice $\mathbb{T}$ with values in S for any $t \in T$ (see [4, 6]). For simplifying notations we shall discuss