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Sommerfeld-Watson Representation for Double-Spectral Functions

I. Potential Scattering without Regge Poles

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Abstract. We demonstrate the existence of elastically unitary scattering amplitudes, corresponding to a given Born term, when the latter satisfies certain conditions of Hölder-continuity. The results are directly applicable to nonrelativistic potential scattering; and they will also be useful later in connection with the construction of relativistic, crossing-symmetric amplitudes. In this preliminary study, no singularities are allowed in the right half of the complex angular-momentum plane; but it is intended eventually to introduce such singularities (Regge poles and branch-points).

I. Introduction

During the years 1968–1970, it was proved [1–3] that there exists an infinity of crossing-symmetric amplitudes that satisfy the Mandelstam representation for pion-pion scattering. These amplitudes satisfied elastic unitary exactly between the elastic and inelastic thresholds, while the inelastic unitary inequality was observed above the latter. Although the full extent of the infinite family of such amplitudes was not investigated, it was established that there is a completely arbitrary contribution to the inelastic double spectral function [1], that there is in general an undetermined subtraction constant [2], and that the Castillejo-Dalitz-Dyson (CDD) ambiguity occurs [3].

Despite the plethora of possible solutions, it is hoped that the Mandelstam representation may be used to build a phenomenological pion amplitude. By injecting sufficient experimental information into the system of equations, it should be possible to restrict the set of solutions to a narrow class, from which other experimental paramaters may be predicted. The work of Basdevant, Froggatt, and Petersen, with the simpler equations of Roy [4], already indicates that such a scheme may well have predictive value.

A serious shortcoming of the work so far is that it has proved impossible to construct amplitudes with full crossing, elastic unitary up to the inelastic threshold, and positivity in the inelastic region, if the elastic