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Phase Transitions of Hard Sphere Lattice Gases*

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Abstract. A unified proof is given for the existence of a phase transition for any two or three dimensional lattice gas with hard cores excluding nearest neighbor occupancy, provided only that the lattice is an open one (possessing two sublattices). It is not required that one sublattice be a translate of the other. Consequently the proof applies to the plane hexagonal and to the diamond lattices, as well as the "cubic" lattices previously proved to have phase transitions. The models are converted to equivalent Ising spin 1/2 systems on one sublattice by a "partial trace" over the other. The spin system has many-spin interactions including some of odd order, which destroys up-down symmetry, but recent work of Pirogov and Sinai on such systems is shown to be applicable and to prove the existence of the phase transition.

I. Introduction

One of the most useful techniques for the rigorous proof of the existence of phase transitions in model systems has been the "contour" method originally due to Peierls [1]. The proof consists in showing that boundary conditions determine the equilibrium state throughout an arbitrarily large system – under conditions of sufficiently large interaction potentials (or sufficiently low temperature). Thus the boundary conditions determine the sign of the magnetization of an Ising system, the density of an attractive lattice gas, or the relative concentrations in a multicomponent system.

In each of these cases the proof succeeds only on a portion of some "symmetry line" appropriate to the model, such as zero magnetic field in the Ising case. For the attractive lattice gas the corresponding symmetry requirement is that the chemical potential (one-body energy) just balance the pair interactions in the completely filled lattice. For the multicomponent Widom-Rowlinson model the symmetry condition is that the chemical potentials of all components be equal [2]. In each case the symmetry condition insures that there is no term in the Hamiltonian proportional to volume that favors any of the competing equilibrium

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