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Interface Sharpness in the Ising System

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Abstract. A simple proof is given for the existence of a sharp interface in three-dimensional Ising systems, at least up to the critical temperature of the corresponding two-dimensional system.

1. Introduction

For the three-dimensional Ising model with nearest neighbour interactions on a simple cubic lattice Dobrushin has shown [1] that at low enough temperature there can exist a *sharp* interface between areas of opposite magnetization. A horizontal sharp interface is characterized by a vertical level l and a positive constant a, independent of the size of the system, such that the expectation values of all spins above l are $>a$ and those of spins below l are $<-a$, or vice versa. The spin system can be forced into a state possessing such an interface by applying a positive magnetic field to all the boundary spins above the level l and a negative magnetic field to all the boundary spins below l.

On the other hand for the two-dimensional square Ising model with nearest neighbour interactions Gallavotti has shown [2] that even at very low non-zero temperature no sharp interface exists. By low-temperature series expansions Weeks, Gilmer, and Leamy [3] found strong evidence that the three-dimensional system has a "roughening-temperature" T_R above which the interface is no longer sharp. For this roughening temperature they find values of about 0.57 times the critical temperature T_c. This is somewhat larger than the critical temperature of the corresponding two-dimensional system, which is about half of the critical temperature of the three-dimensional system.

Here we give a very simple proof that the critical temperature of the two-dimensional system is a lower bound on the roughening temperature T_R. We use a variant of Percus' method of the "duplicate set of variables", which has recently been described by Lebowitz [4][1]. Moreover we will

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