

# Decay Properties and Borel Summability for the Schwinger Functions in $P(\Phi)_2$ Theories

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**Abstract.** For the truncated Schwinger functions of the  $P(\Phi)_2$  field theories, we show strong decrease in the separation of points. This shows uniqueness of theories with  $P$  of degree four. We also extend the domain of analyticity in the coupling constant. For theories with  $P$  of degree four, the combination of these two results gives Borel summability.

## Introduction

In this paper, we consider the two dimensional Euclidean boson field theories and we give bounds on the truncated Schwinger functions which have the decay properties expected from perturbation theory and introduced in statistical mechanics in [3]. We use methods known from statistical mechanics [9] to obtain these bounds.

We first formulate the bound and give then some applications. The Schwinger functions for Euclidean field theories in two dimensions in a finite (space-time) volume  $\Lambda$  are defined as the moments of the normalized measure

$$e^{-\lambda V(\Lambda)} d\mu_{m^2} / \int e^{-\lambda V(\Lambda)} d\mu_{m^2},$$

where  $d\mu_{m^2}$  is the Gaussian measure on  $\mathcal{S}'(\mathbb{R}^2)$  with mean zero and covariance  $(-\Delta + m^2)^{-1}$ , and

$$V(\Lambda) = \int_{\Lambda} d^2x : P(\Phi) : (x).$$

Here,  $P$  is a lower bounded polynomial, and Wick ordering  $::$  is with respect to the free theory defined by  $d\mu_{m^2}$ .

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