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Infinite Volume Asymptotics in $P(\phi)_2$ Field Theory

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Abstract. We prove a number of asymptotic results in the $P(\phi)_2$ theory in the limit when the space cut-offs are removed, in particular the behavior of E_l and $Z_{t,l}$ as $t, l \to \infty$. Such results are used to study the question of orthogonality of infinite volume Euclidean measures $\mu_\infty(\lambda)$ for varying interaction constants λ.

1. Asymptotics

In this paper we consider any fixed real polynomial $P(y)$ with $P(0)=0$ which is bounded below, and the corresponding $P(\phi)_2$ quantum field theory in two-dimensional space-time [1]. The approximate, or cut-off, Hamiltonian is

$$H_l = H_0 + \lambda \int_{-l/2}^{l/2} :P(\phi(x)): dx \tag{1.1}$$

where H_0 is the usual free Hamiltonian of mass $m_0 > 0$, and $\lambda \geqq 0$ is the coupling constant. H_l has a simple eigenvalue E_l at the bottom of its spectrum, with corresponding eigenvector Ω_l, the (approximate) physical vacuum. The positive operator $H_l - E_l$ has no spectrum in some interval $(0, m_l)$ where $m_l > 0$. With Ω_0 denoting the bare vacuum in Fock space, it is known that $(\Omega_0, \Omega_l) \neq 0$. Thus $|(\Omega_0, \Omega_l)|^2 = \exp(-l\eta_l)$ defines η_l, where Ω_0 and Ω_l are both taken to have norm 1. The quantity

$$Z_{t,l} = e^{G_{t,l}} = (\Omega_0, e^{-tH_l}\Omega_0) \tag{1.2}$$

is the analogue of the partition function in classical statistical mechanics.

The following asymptotic results are known to hold for any $\lambda \geqq 0$ [2, 3].

Theorem 1. *There are functions $\alpha_\infty(\lambda)$ and $\beta_\infty(\lambda)$ such that*

i) $E_l = -\alpha_\infty l - \beta_\infty + o(1)$ *as* $l \to \infty$.

ii) $0 < A \leqq \eta_l \leqq B < \infty$ *as* $l \to \infty$.

iii) $G_{t,l} = \alpha_\infty t l + o(tl)$ *as* $t, l \to \infty$.