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Tilting at Cosmological Singularities

C. B. Collins

Department of Astrophysics, University of Oxford, Oxford, and Trinity College, Oxford, England

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Abstract. A detailed investigation is made of the simplest type of general relativistic perfect fluid cosmological models that possess a singularity at which all physical quantities are well-behaved. These models are spatially homogeneous, axisymmetric generalisations of the open ($k = -1$) Robertson-Walker universes. A pictorial description of the evolution of the models is obtained by using the qualitative theory of differential equations.

The most surprising feature that emerges is that for some (non-empty) models the matter density may become zero, within a finite time, on a null hypersurface which acts as a Cauchy horizon for the models. This result is generalized to most other types of spatially homogeneous models.

It is also discovered that the behaviour of the models varies dramatically with the type of matter content. This casts some doubts on the validity of assuming definite equations of state in general relativity, and suggests an investigation of the structural stability of Einstein's field equations.

1. Introduction

Ever since the discovery that the spatially homogeneous and isotropic Robertson-Walker models of the universe possess a "big-bang" singularity, at which the density of matter and the curvature of space-time become infinite, conventional cosmology has seized the opportunity to interpret it as a theoretical triumph heralding the birth of our universe. Moreover, any suspicions that the existence of the singularity is a consequence entirely of the very high symmetry of the models have since proved to be unfounded. Powerful theorems, established chiefly by Hawking and Penrose ([1], and references cited therein), require that, under certain reasonable conditions, any sufficiently general solution of Einstein's field equations of general relativity will possess a singularity; subsequent work of Ellis, Sciama, and Hawking [2, 3] shows that our universe will indeed satisfy the requirements for the theorems to hold.

However, the type of singularity that the Hawking-Penrose theorems entail is more general than the big-bang sort. Rather more subtle concepts, relating to the inextendibility of causal (timelike or null) curves in the space-time manifold have had to be introduced.