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Markovian Master Equations

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Abstract. We give a rigorous proof that under certain technical conditions the memory effects in a quantum-mechanical master equation become negligible in the weak coupling limit. This is sufficient to show that a number of open systems obey an exponential decay law in the weak coupling limit for a rescaled time variable. The theory is applied to a fairly general finite dimensional system weakly coupled to an infinite free heat bath.

§ 1. Introduction

In the last fifteen years there has been a growing realisation by physicists of the importance of master equations for the study of the time evolution of open quantum-mechanical systems. As well as providing a suitable framework for the consideration of the fundamental property of irreversibility [1], they have proved an important technique in the analysis of a variety of models, such as harmonic oscillators and lasers. It becomes clear in the excellent survey article of Haake [2] that one of the main reasons for the usefulness of master equations is the radical simplification obtained when memory effects are neglected.

It is rather surprising, therefore, that in the recent rigorous studies of these models, the use of master equations has been avoided. This appears to be because, although it is possible to give a rigorous proof of the master equation itself, conditions under which the memory effects can be neglected have not been found.

In this paper we give a rigorous proof that the time evolution of an open system is Markovian in the weak coupling limit. As the coupling constant converges to zero we rescale the time variable to compensate for the slower decay of the system. The theory is developed in a general form and its application to a variety of models is outlined. The case of a general finite-dimensional system weakly coupled to an infinite free heat bath is investigated in some detail and relaxation to the Gibbs state is proved.