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The Mass Gap for the $P(\phi)_2$ Quantum Field Model with a Strong External Field

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Abstract. We consider the $P(\phi)_2$ hamiltonian whose interaction density is given by

$$\lambda P(\phi(x)) + \mu\phi(x)^k$$

where k is odd and $1 \leqq k < \deg P$. For sufficiently large μ we show that there is a gap in the energy spectrum. In addition we obtain new regions of analyticity in λ and μ for the Schwinger functions and the pressure.

§ 1. Introduction

The $P(\phi)$ quantum field hamiltonian in two space time dimensions is given formally by

$$H = H_0 + \lambda \int :P(\phi(x)): dx - E \geqq 0 \tag{1.1}$$

where H_0 is the free hamiltonian of mass m_0, P is a positive polynomial and E is the vacuum energy. In [2, 3], it was shown that for sufficiently weak coupling (λ/m_0^2 small) the vacuum (ground state) of H is unique and that the mass of H is positive. In this paper we consider the Hamiltonian

$$H_\mu = H_0 + \int :P_\mu(\phi(x)): dx - E_\mu \tag{1.2}$$

where

$$P_\mu(\xi) = \lambda P(\xi) + \mu \xi^k \tag{1.3}$$

and k is odd, $1 \leqq k < n \equiv \deg P$. Our main result is that if μ is sufficiently large then the mass of H_μ is positive. We also show that the infinite volume Schwinger functions are analytic in λ and μ, provided $|\lambda|$, $|\mathrm{Im}\,\mu|$ are small, $\mathrm{Re}\,\lambda > 0$ and $|\mu|$ is large. The results of [2] can also be obtained for large μ.

If $P(\xi) = \lambda\xi^4 + \mu\xi$, $\mu \neq 0$, Simon and Griffiths [10] have established uniqueness of the vacuum and analyticity of the pressure (vacuum energy per unit volume) as a function of μ for $\mathrm{Re}\,\mu > 0$. Their proof follows

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